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## **Modelling of Currency outside Banks in Croatia**

### **Abstract**

The cooperation of two organisational units of the CNB on the improvement of the forecasting methodology used by the CNB resulted in the development of two econometric models for the short-term projections of currency outside banks. The first model is the deterministic model that is describing weekly, monthly and annual seasonality . The second model, together with deterministic seasonality assumes the ARIMA structure of the residuals. Both models outperform the existing forecasts done in the CNB.

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## 1 Introduction

With the change of the use of monetary policy instruments in 2005 that gave the prominent role to the open market operations (OMO), a need has arisen for the strengthening the liquidity management framework by the Croatian National Bank. It was necessary to modify and improve the existing daily liquidity forecasts as the key element of the liquidity management. The structure of the daily liquidity forecasts was changed in such a way that it now provides information in the form that simplifies a deciding how much liquidity needs to be created by regular open market operations in order to reduce the volatility of the money market interest rate.

The daily liquidity forecast depends on the accuracy of its individual components. For that purpose the Liquidity Forecasting Division of the Monetary Operations collaborated with the Econometric Modelling Department on estimation of time-series econometric models for several components of the liquidity forecast. This paper describes the forecasting models for currency outside banks.

The second section of the paper briefly describes the process of the liquidity forecasting in the CNB. The third section describes the forecasted series of currency outside banks in Croatia. The time series models developed for the purpose of forecasting currency outside banks and the evaluation of the forecasts obtained by these models are described in the fourth section of the paper. The final section concludes.

## 2 Liquidity Forecasting in the CNB

As the main motive for starting the open market operations is to mitigate the volatility of excess holding of reserves with the CNB by the commercial banks and thus the volatility of the overnight money-market interest rates, the new structure of liquidity forecast facilitates the assessment of the amount of funds that the Croatian National Bank needs to add to or withdraw from the system through its regular open market operations. Namely, the additional level of liquidity that the central bank must provide for the equilibrium in the market depends primarily on the difference between the supply and demand for bank reserves.

The short-term liquidity forecast uses the entire balance sheet of the CNB;

however, the creation of the liquidity forecast starts with the autonomous factors that determine the autonomous supply of the bank reserves. The autonomous factors are outside of the CNB's control in the short run and they include net foreign assets, currency outside banks, government deposit in domestic currency, and other autonomous factors. Net foreign assets, the largest autonomous factor, and other autonomous factors (other items net) primarily depend on the exchange rate, but this effect mutually cancels (as NFA increases when exchange rate depreciates, the OIN decreases and vice versa). On the other hand, the changes in holding of the currency outside banks and in the government deposits in domestic currency do have a strong influence on the increase or decrease of the domestic currency liquidity of the banking system. Whereas the government deposit is the most unpredictable item in the liquidity forecast<sup>1</sup>, the currency outside banks shows very regular patterns with strong seasonal characteristics. Thus, it is possible to accurately forecast the short-term movement of currency.

The Banking System Liquidity Forecasts Section collects, processes, and analyze the information on domestic liquidity and prepares forecasts of various maturity within one period required reserves maintenance. The short-term forecasts are updated on a daily basis using the monthly and weekly behavioural patterns exhibited in the past together with the most recent data (preliminary data of currency outside banks are available with one day lag). The combination of the expert knowledge and historical data is used for assessing the effects of holidays on the forecasted items, where the position of holidays within a working week or month is also taken into account. The daily forecast also uses the information on its annual change together with seasonal characteristics of a particular month and intra-monthly seasonality of the series.

### 3 Properties of the Currency Outside Banks Series

The currency outside banks, with a stable trend and strong seasonal factors can be accurately forecasted in the short-term and thus also can be the total liquidity. Any increase in the currency reduces the level of the liquidity in the system and vice versa: any decrease in the currency increases the level of liquidity in the

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<sup>1</sup>The government deposit in the domestic currency was also modelled but results were much weaker than those for the currency outside banks.

system.

The currency outside banks comprises of all banknotes and coins in the national currency that the economic subjects (households, companies and non-residents) keep at a certain moment for transaction purpose or as a store of value. It includes all banknotes and coins of the national currency ever issued that are outside of the banking system, i.e. that are available, at a given time, in the non-banking sector. When currency is returned to banks, it is considered to be a part of banks' reserve with the CNB. From the perspective of commercial banks, the increase of currency in their vaults represents the increase in the reserves with the CNB. Thus, the daily calculation of the currency outside banks is performed in the following way:

<b>Total printed banknotes and coins of the national currency</b>	
-	Cash in the Croatian National Bank vaults
-	Cash in transport
-	Cash in FINA and Hrvatska pošta (Croatian Post) vaults
-	Cash in commercial banks vaults
=	<b>Currency outside banks</b>

Figure 1 shows the currency outside banks from November 1998 to January 2006. There is an increasing trend in the currency during that period, which can be attributed to the growing demand for cash in the conditions of the economic growth. The positive impact of the economic growth on the currency is also suggested by the stagnation of the currency during the 1999 recession, and restoration of its growth from the second quarter of 2000, which coincides with the economic recovery. The currency strongly increased at the end of 2001, when, before the introduction of euro banknotes, many Croatian citizens exchanged the banknotes of legacy currencies (DEM, ITL, ATS etc.) into HRK banknotes, which resulted in the permanent increase of domestic currency. The crisis in Riječka Banka in early 2002 triggered the bank run and corresponding withdrawal of household savings from the banking system, which additionally increased the amount of currency outside banks. Since mid-2002, the currency outside banks exhibited a regular pattern with a strong seasonality. The seasonality of the currency series can be seen in the pronounced maximums (peaks) and minimums (bottoms), i.e. seasonal patterns that were identical in several years. The annual maximums of currency

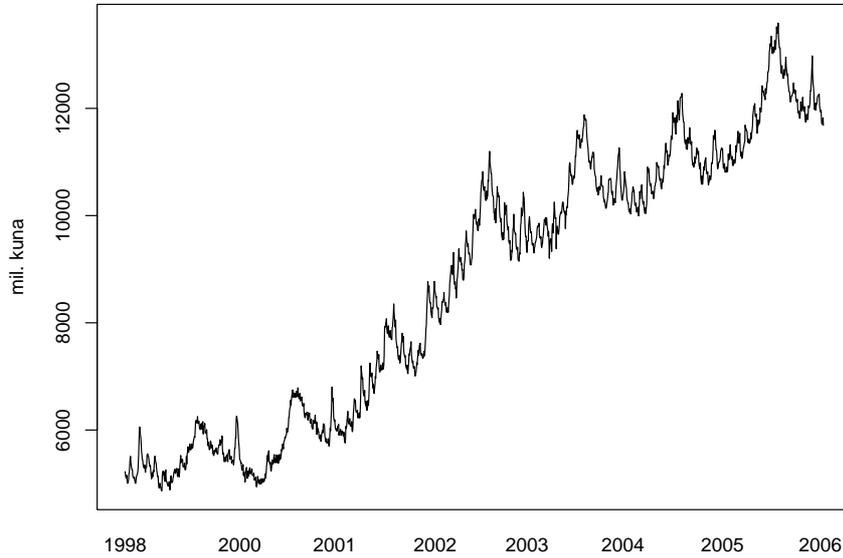


Figure 1: Currency outside banks in the period November 1998 - January 2006

were reached in July and August and at the end of December, whereas the minimums occurred in March and April. A gradual increase in July and August is the result of the summer tourist season, when foreign tourists increase the holding of currency by their demand for kuna. On the other hand, the peaks before Easter holidays and especially before Christmas and New Year are the result of the demand for cash by domestic population during the periods of high domestic consumption.

*Table 1 Basic Statistics of Currency Outside Banks (mil. kn)*

	1999.	2000.	2001.	2002.	2003.	2004.	2005.
<i>Average</i>	5498,4	5774	7021,8	9430,6	10361,7	10895,1	11960,7
<i>Std. dev.</i>	334,1	535,2	701	782,9	666	547,3	715,2
<i>Max</i>	6258,5	6803,3	8767,7	11198,7	11879,3	12279,8	13592,1
<i>Min</i>	4863,2	4939,2	5758,1	7967	9202,5	9994,3	10789,4
<i>Beginning of the year</i>	5588,4	5808,2	6247,6	8403	9605,8	10508,5	10872,7
<i>End of the year</i>	5827,1	6592,9	8507,4	9680,9	10573,1	10955,6	12307,1

The periods of high level of currency are followed by its decrease in September, and in January and February. The currency reaches its annual minimums in the period from January to April and in November to early December.

This pattern of currency outside banks is used when forecasting currency at the annual basis. In the preparation of a forecast, the extreme points are set, taking into account the expectations about the tourist season that raises the summer maximum, the consumption during Christmas shopping season and the payout of annual bonuses that increase the winter maximum, as well as the information about increases in wages and pensions. The effect of the increased consumption before Easter is not very significant as the above mentioned maximums, especially since the first months of the year are the periods with a lower level of the cash outside banks.

Although the starting point when forecasting the currency is its annual level, the main task of the Liquidity Forecasts Division is to forecast the currency within one month, or within one week. Namely, the daily information is the most important for the liquidity management and it is obtained through monthly and weekly projections.

For the forecast of the monthly level of currency outside banks, we are using the database maintained since early 1997, which shows its annual patterns. The daily changes of the cash are forecasted on the basis of daily changes in the previous month of the current year and the daily changes in the same month of the last year. Information about the major payments in the month are also taken into account: the payments of salaries in the public sector, payments of pensions through bank accounts and Hrvatska pošta (Croatian Post), and extraordinary payments, such as payment of old savings, vacation bonuses and Christmas bonuses. For the sake of better understanding, the movement of currency within one month should be divided into two intervals: the first interval when the currency outside banks is gradually increasing from the 5th to (approximately) the 15th of the month, due to the payment of salaries and pensions. Here it must be noted that the cash also increases slightly at the beginning of the month, and around the 10th of the month the level of currency starts its rapid increase, reaching the maximum between the

15th and the 20th of the month. After this, in the second interval, the currency starts dropping and constantly decreases as households spend their salaries and pensions and there is no additional income in that period.

However, in order to explain the movement of currency within a month, we must understand the calculation of currency outside banks as showed in Figure 1. The amount of currency in FINA <sup>2</sup> indicates the increase or decrease of the cash held by households, i.e. the payments of salaries and pensions as well as the household consumption. The amount of currency in Hrvatska pošta indicates the payout of pensions in cash through postal system. The payment of salaries and pensions has the strongest effect on the level of currency within a month (with the exception of summer season and days before holiday seasons). The payment of pensions through the postal system is conducted in cash, and its amount does not depend on the recipient's preference to either keep the funds in a bank or withdraw it as cash. The government monthly pays salaries amounting to more than 1 billion kuna, which is usually done by the 15th of the month. The monthly amount of pensions paid out through banks is additional billion kuna which is made between the 10th and the 12th of the month. The monthly amount of pensions paid out through the postal system is about 500 million kuna, and is paid out between the 13th and the 15th of the month. The payment of salaries and pensions significantly increase the level of currency outside banks. However, the uncertainty about the start of the payment of pensions was one of the largest problems when forecasting the currency outside banks, and those payments were very frequently delayed in the past, so the currency could not reach its maximum before the 19th or the 20th of the month.

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<sup>2</sup>former Croatian clearing house, the agency that distributes cash on the behalf of the CNB

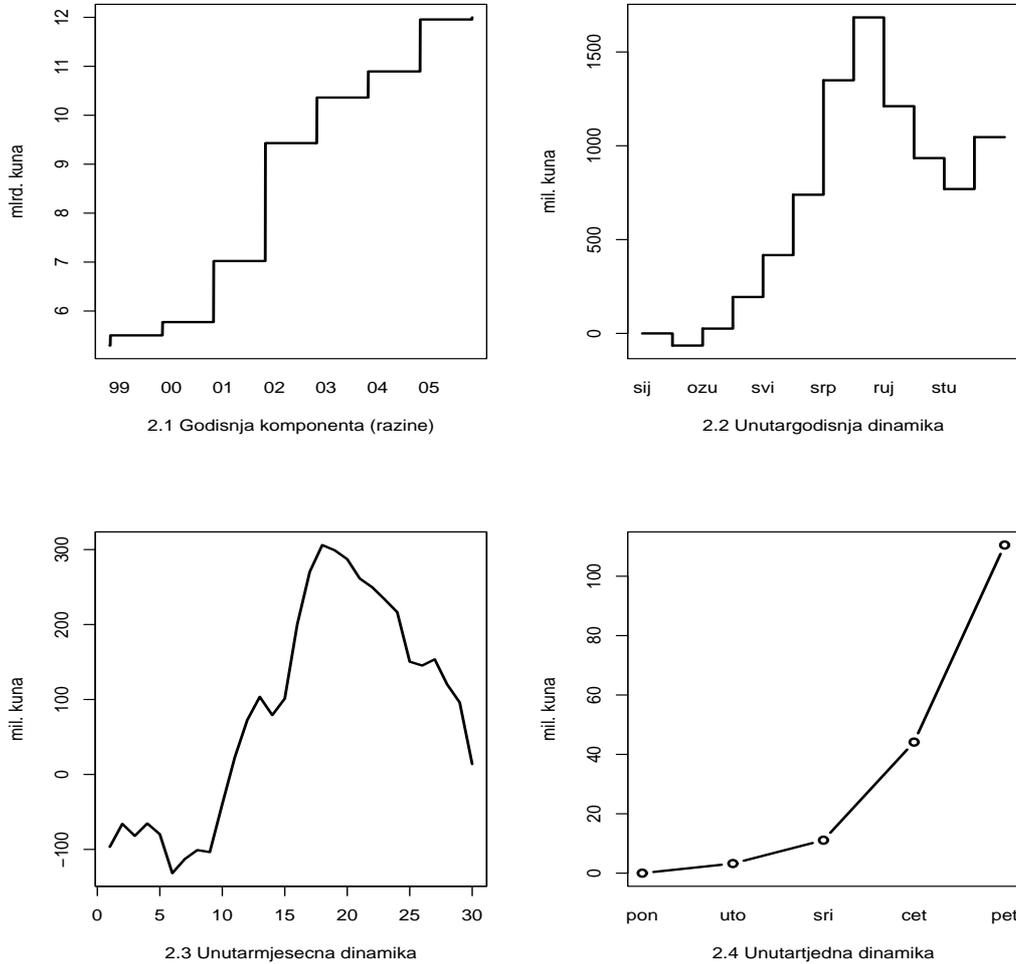


Figure 2: The figure shows the seasonal patterns of currency outside banks. Figure 2.1 provides the annual averages of currency outside banks. There was a significant increase in currency in 2002, which can be attributed to two main reasons: the introduction of euro banknotes and crisis in Riječka banka. Figures 2.2, 2.3 and 2.4 show effects of a particular month of the year, day of the month and the day of the week on the total level of the currency outside banks, presented as the deviation from January, the 31th day of the month and Monday respectively. The method according to which the series has been divided into seasonal components (seasonal factors) is similar to the movement of currency model in the following chapter, but is applied to the original (non-differentiated) movement of currency series.

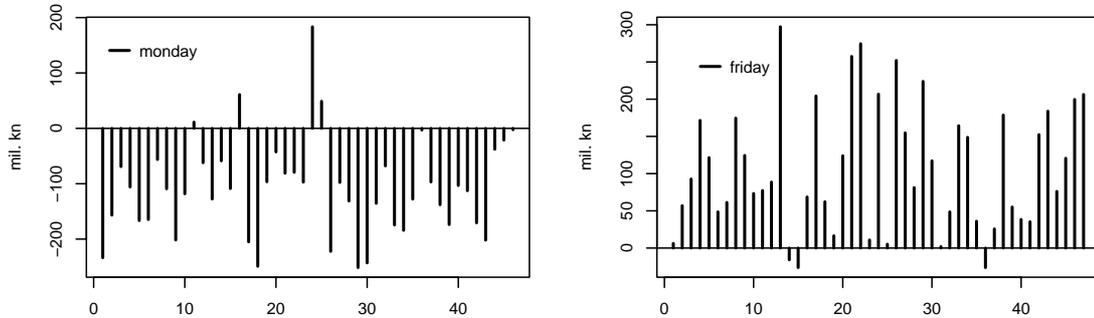


Figure 3: Changes of currency outside banks on Mondays and Fridays in 2005.

The regularity of the movement of currency within a week affects primarily the daily updates of the liquidity forecast. In most cases, the movement of currency increases on Fridays and decreases on Mondays. This can be explained by the preference of the population to withdraw cash for shopping over the weekend, and the amount of cash outside banks decreases on Monday when stores returned cash received during weekend to FINA, which is shown in Figure 3. In the middle of the week these factors are not so strong, but the decreasing tendency is still present on Tuesdays, and the currency increases slightly on Thursdays. The intensity of the increase or decrease of the currency over the weekend depends also on the position of the particular weekend within the month (first, second, third, fourth or fifth weekend of the month).

#### 4 Modelling the Currency Outside Banks Using the Time Series Analysis

The time series of currency outside banks with its regular patterns and a large number of observations is an ideal candidate for the application of time series analysis for forecasting of its future values. Time series analysis has been widely used in economic forecasting during the last twenty years, especially for forecasting the near future. Many central banks use similar models in their currency forecasts.

As prior to this project the CNB did not use any formal statistical model for forecasting the daily series of currency outside banks, the first step was to set up a simple model that resembles existing process of making projections manually as described in the previous section. This model (Regression model (REG)) describes the dynamics of the currency outside bank in the form of multiple linear regression using only deterministic explanatory variables. After that we applied the standard ARIMA methodology to develop the second model that consists of the regression model and the seasonal ARIMA model.<sup>3</sup>

Both models are to be used for forecasting of the currency outside banks. In order to create accurate forecasts, it is necessary to formalize its annual, monthly and weekly patterns. In addition, we would like to model effects of holidays on currency holding. Finally, we eliminate long term (nonstationary) component by differencing the series.

#### 4.1 The regression model of the currency outside banks

The first model, expressed in the first differences, is the following:

$$\Delta y_t = \sum_{i=1}^{12} \alpha_i M_{it} + \sum_{i=1}^4 \beta_i TD_{it} + d_t + \sum_{i=1}^k \Theta_i(B) \delta_{it} + \sum_{i=1}^j \gamma_i O_{it} + \varepsilon_t, \quad (1)$$

where  $y_t$  is the level of currency outside banks at time  $t$ ,  $\varepsilon_t$  is the error term at time  $t$ , and deterministic variables  $M$ ,  $TD$ ,  $d$ ,  $\delta$  and  $O$  describe seasonal effects in the following way:

**Intra-weekly effect (TD)** For each weekday  $i$  ( $i = Mon, \dots, Fri$ ) we define a dummy variable that at time  $t$  has a value 1 if the day at time  $t$  is day  $i$  or 0 otherwise

$$TD_{it} = \begin{cases} 1, & \text{if day } i \text{ occurs at time } t \\ 0, & \text{otherwise.} \end{cases}$$

**Monthly effect (M)** In the model we assume that for each calendar month changes of currency in circulation are stable, but on different levels. According to that, for each calendar month  $i$  ( $i = Jan, \dots, Dec$ ) we define :

$$M_{it} = \begin{cases} 1, & \text{if month } i \text{ occurs at time } t \\ 0, & \text{otherwise.} \end{cases}$$

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<sup>3</sup>A. Cabrero et al. and Hlavaček et al. (2005)

**Intra-monthly effect (d)** Intra-monthly seasonality is conveniently defined as a linear combination of trigonometric functions:

$$d_t = \sum_{j=1}^p \left( a_j \sin \frac{2j\pi m_t}{M_t} + b_j \cos \frac{2j\pi m_t}{M_t} \right),$$

where  $m_t$  stands for the day of the month and  $M_t$  is the total number of days of a given month. Parameter  $p$  defines the number of different frequencies that we use in modeling the intra-monthly dynamics, but they need to be used sparsely. Alternatively, we could model intra-monthly effect with dummy variables for each day of the month (the total of 31 variables).

**Holidays ( $\delta$ )**  $\delta_i$  is the indicator function of holiday  $i$ :

$$\delta_{it} = \begin{cases} 1, & \text{if holiday } i \text{ occurs at time } t \\ 0, & \text{otherwise.} \end{cases}$$

$\Theta_i = \Theta_i(B)$  is polynomial in variable  $B$ , where  $B$  is the standard backward shift operator. Term  $\Theta_i(B)\delta_{it}$  should capture the change of the currency level around holiday  $i$ .

**Outliers** In the analysis of the residuals, largest outliers are identified and their possible effect on other parameters is removed with dummy variables. Formally, if there is unexplainably high residual at time  $t$ , we define the following dummy variable:

$$O_{it} = \begin{cases} 1, & \text{if outlier } i \text{ occurs at time } t \\ 0, & \text{otherwise.} \end{cases}$$

We estimate regression coefficients  $\alpha_i, \beta_i, a_i, b_i, \gamma_i$ , coefficients of the polynomial  $\Theta_i(B)$  and residual variance. The estimation procedure was performed by applying the ordinary least squares method.

## 4.2 The ARIMA model of currency outside banks

The second model we used has the following structure:

$$y_t = \sum_{i=1}^4 \beta_i T D_{it} + d_t + \sum_{i=1}^k \Theta_i(B) \delta_{it} + \sum_{i=1}^j \gamma_i O_{it} + \eta_t, \quad (2)$$

Regression variables are defined as in the previous model, but now we assume that the residuals have ARIMA structure:

$$\eta_t = \frac{\theta(B)}{\phi(B)\delta(B)}\varepsilon_t,$$

where  $\theta(B)$  and  $\phi(B)$  are autoregressive and moving-average polynomials with all zeros outside the unit circle, and  $\delta(B)$  is a difference polynomial (with all zeros just on the unit circle).

Model (2) consists of the regression and the ARIMA model, so this class of models are sometimes called regARIMA models. Deterministic component describes intra-weekly and intra-monthly effects, while stochastic component describes correlation structure of the series and describes the remaining periodicity.

In our analysis, the chosen ARIMA model must fulfill some standard criteria: stability of parameters, good in-sample characteristics, minimal autocorrelation of residuals, and, most importantly, model needs to have good forecasting properties.

Analysing the sample from 1999-2004 we chose the following ARIMA polynomials in the model of the residuals<sup>4</sup>:

$$\begin{aligned}\delta(B) &= (1 - B^{261})(1 - B) \\ \theta(B) &= 1 + \theta_1 B^6 + \theta_2 B^7 + \theta_3 B^{10} + \theta_4 B^{12} + \theta_5 B^{14} + \theta_6 B^{21} + \theta_7 B^{22} + \theta_8 B^{65} \\ \phi(B) &= 1 - \phi_1 B^{261}\end{aligned}$$

Bell and Hillmer (1983.) suggested the following procedure for estimation of parameters in such models. First, by analyzing the autocorrelation function (SACF) and function of partial autocorrelation (SPACF) of residuals one can define the level of regular ARIMA residual and eventual need for seasonal differencing. After differencing the series, regression and ARIMA parameters are to be estimated simultaneously using maximum likelihood method. Parameters can also be estimated using nonlinear least squares since they are asymptotically equal to the ML estimates. In this paper we used nonlinear least squares.

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<sup>4</sup>The series of currency outside banks is available for working days only. Data for weekends and holidays are not available. In order to mitigate this problem only working days were used. The missing data for holidays that fell within a working week were constructed as an average of neighboring working days. There are approximately 261 working days in a given year, so differencing at 261 lags removes the most of the annual seasonality. 65 represents quarterly seasonality, and 21-22 monthly seasonality.

## 4.3 The evaluation of forecasts

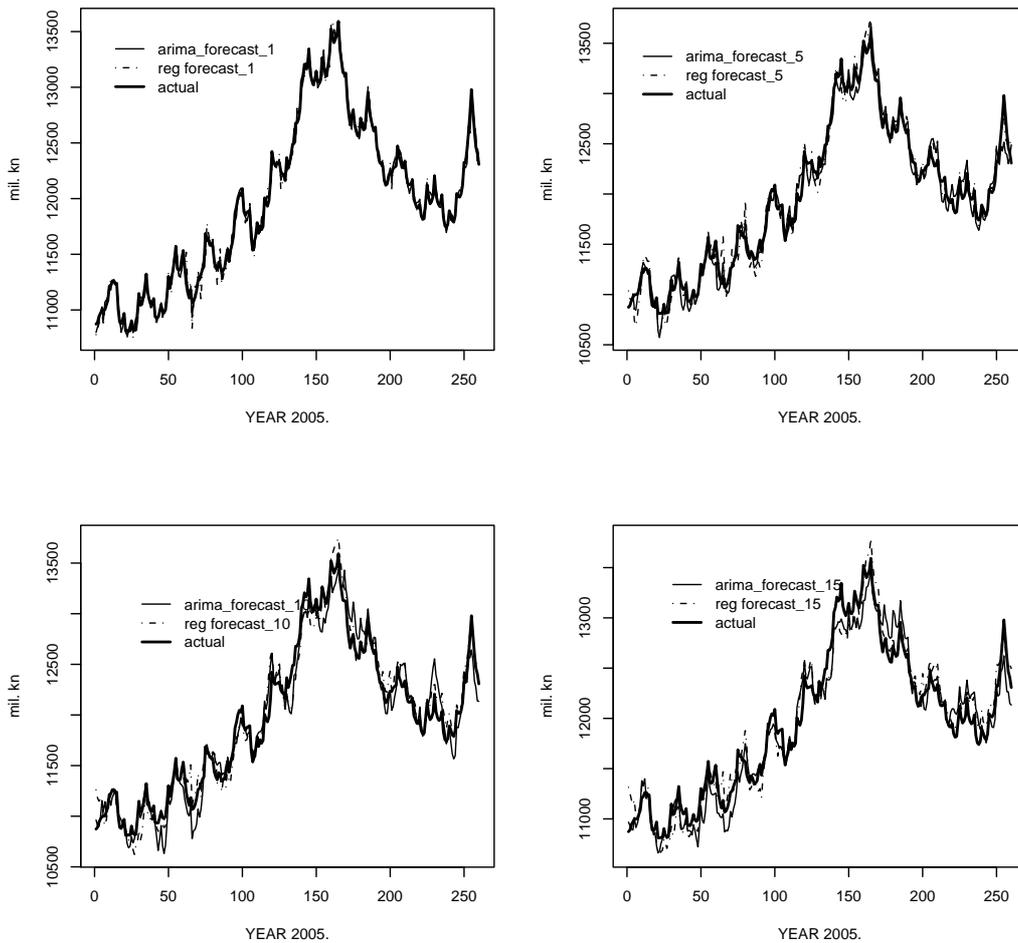


Figure 4: Forecasts of currency outside banks with different horizons

The main purpose of the described models is to forecast the future values of currency outside banks, so it is important to evaluate their out-of-sample forecasts. For that purpose, models' parameters were estimated on the sample from August 2002 to December 2004. Their out-of-sample forecasts for 2005 are then compared with actuals. Procedure was such that both models were reestimated with each new observation, and forecasts were stored. The series of forecasts were then used to calculate the forecast errors in order to choose the best model.

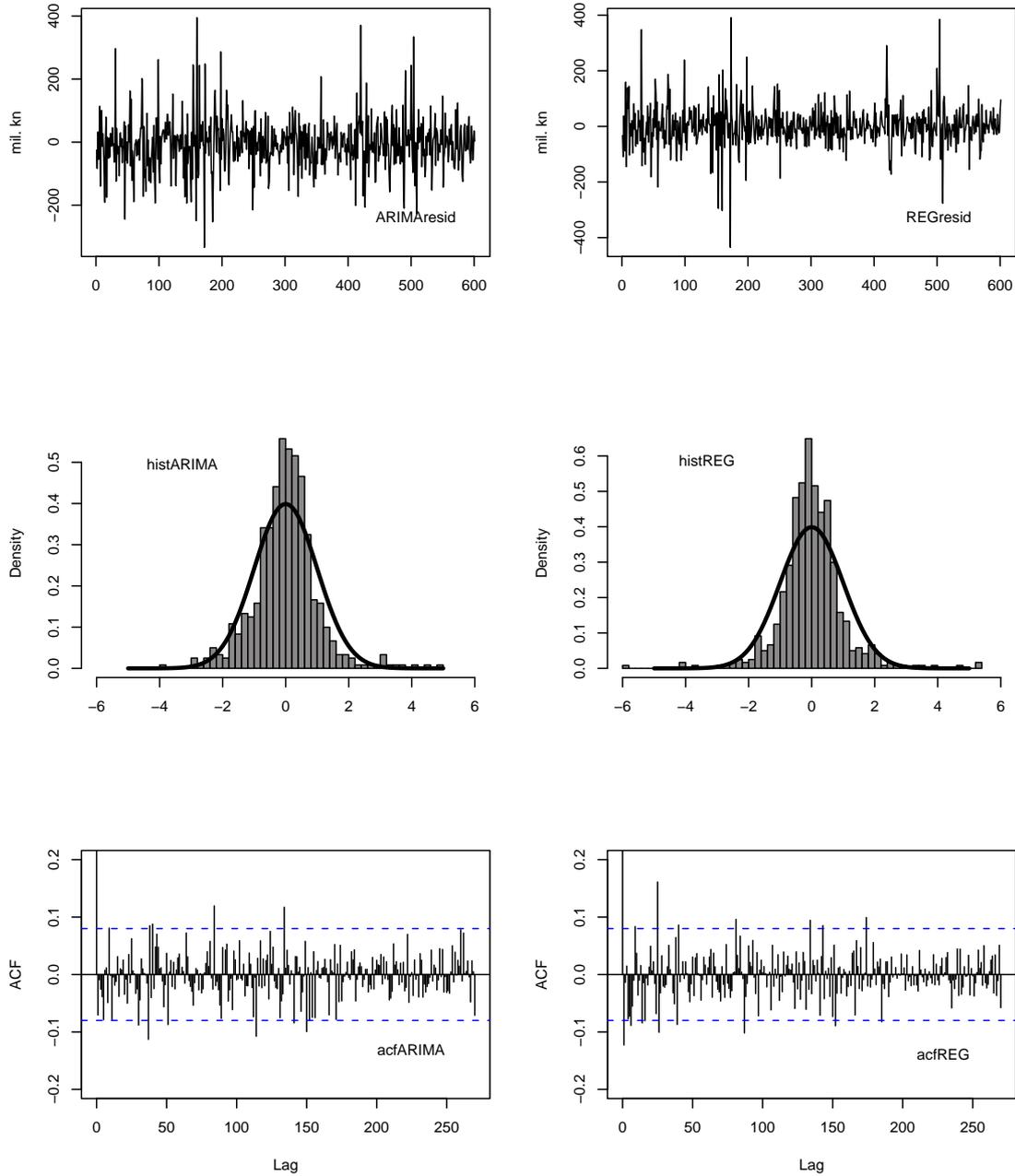


Figure 5: Analysis of the residuals from the estimated models

Models described in previous section, the Regression model (REG) and the ARIMA model (ARIMA), are compared with the Naive model (Naive) which assumes no change in the level of currency in the future<sup>5</sup>, as well as with the forecasts created by the Liquidity Forecasting Division of the CNB (*Expert*).

Table 2 Evaluation statistics for regression model (REG), ARIMA model (ARIMA), official model (*Expert*) and random walk model.

<i>h</i>	<i>MAE (mil. kn)</i>				<i>RMSE (mil. kn)</i>			
	REG	ARIMA	Expert	RW	REG	ARIMA	Expert	RW
1	44.71	53.28	52.6	80.89	56.94	70.54	70.8	106.41
2	62.94	75.15	79.4	123.65	79.93	95.19	102.6	157.01
3	73.93	82.71	/	153.67	94.46	106.52	/	191.34
4	82.13	90.14	/	176.63	103.25	116.22	/	216.00
5	89.28	91.68	117.4	199.10	113.69	118.54	160.7	239.55
6	95.77	91.46	/	229.10	122.65	118.48	/	281.66
7	100.97	96.19	/	254.93	129.27	122.93	/	314.18
8	106.84	97.32	/	275.10	134.43	123.18	/	336.68
9	112.49	100.37	/	286.71	139.84	127.37	/	351.28
10	119.11	102.90	/	299.70	146.53	133.38	/	365.85
15	124.01	112.02	/	318.85	157.06	140.01	/	407.83
20	137.98	129.77	/	359.54	167.83	154.69	/	430.82

Standard evaluation of statistics shown in the Table 2 shows that both models outperform the Naive (random walk) model, due to strong seasonality of the series. However, the actual quality of the estimated models can best be assessed by comparison with the forecasts created by the CNB staff as described in the first two sections. It is important to emphasise that staff projections were based on preliminary data on currency, which is different from the final data. Thus the total error in the staff forecasts of currency outside banks includes both the standard forecast error and the difference between the preliminary and final series.<sup>6</sup>

Expert model thus constantly gives somewhat worse forecasts than the regression model and the ARIMA model. Exceptionally, one day ahead forecast with the ARIMA model has slightly larger mean absolute error than the Expert model,

<sup>5</sup>Here we assume simple random-walk structure of the currency outside banks:

$$y_t = y_{t-1} + \varepsilon_t.$$

<sup>6</sup>See Appendix A

but for longer forecasting horizons both statistics used for evaluating forecasting performance (MAE, RMSE) show that estimated models outperform the Expert model.

The results suggest that the Regression model (REG) gives the best short-term forecasts - up to 5 days ahead - while ARIMA model outperforms it at longer horizons. However, forecasts are created on a large time interval (the entire year) and the model with lowest forecasting error for the entire year, does not necessarily create the best forecasts throughout all the subintervals of that year. When applying this model, it is important to examine in more details the forecasting performances of both models for each calendar month, around holidays, etc.

Consistent with the regular characteristics of projections based on autoregressive models, longer period ahead forecasts created by the ARIMA model have larger forecasting errors than on shorter term horizons. That is so because the forecasting error is being accumulated, i.e. the two day ahead forecast by definition includes the one day ahead forecast (and its error term). Similarly, three days ahead forecast is being constructed based on two days ahead forecast, and the same procedure is used when forecasting more distant future. This forecasting method is called chain forecasting and is standardly used in the models with differences or in the autoregressive models.

Figure 4 shows actual series of currency outside banks and its forecasts generated with the described models. Forecasts are very good for shorter horizons, while at horizons of fifteen or more days the errors are more pronounced, especially during the summer months.<sup>7</sup>

Figure 5 shows in-sample residuals of both models. There are few outliers whose influence can be removed with dummy variables as described in (4.1). Histograms of standardized residuals of both models do not resemble the Gaussian density very well, but this can be somewhat corrected by proper treatment of outliers. Sample autocorrelation functions suggest that there is still some autocorrelation left in the model.

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<sup>7</sup>A possible explanation of larger forecasting errors during the Summer of 2005 can be found in very good tourist season and thus also demand for cash

## 5 Conclusion

This paper describes two simple time series models for forecasting the currency outside banks in Croatia. Both models produce good short term forecasts, and outperformed the forecasts done by the CNB staff during 2005. The models complement each others in the sense that the first model, the Regression model based on the first differences, outperforms the second model, the ARIMA model, on the intervals up to 5 days ahead. On the other hand, the ARIMA model outperforms the Regression model at longer horizons. Due to differences in the approach of those two models, it is to be expected that one can find a combination of their forecasts that would outperform both models. This combination forecast might use different weights for each calendar month. Application of both models in regular liquidity forecasting framework should thus both improve and ease the production of daily forecasts of currency outside banks. The results should still be treated with caution and analysts who prepare the daily forecast need to evaluate the model forecast against overall economic situation and adjust it for expected outliers in the series, such as one time payments of debt to pensioners etc.

An earlier version of the first model has been used by the Liquidity Forecasting Division since June 2005 and initial experience has been positive. It was good with the forecasting horizon of up to 10 working days, although it failed to fully anticipate some sudden changes in the level of currency, especially during the Easter 2006. The proposed models should improve the accuracy of the forecasts.

There are still a lot of improvements to be made in the daily forecasting framework of the CNB. On one hand, a variety of more advanced forecasting methods such as Kalman filters could be employed. On the other hand, there are other variables (autonomous factors) such as cash in the commercial banks' vaults and government deposits whose forecasts can be further improved using the methods described in this paper.

## A Difference between preliminary and final data on currency outside banks

The daily short-term forecasts are done using preliminary data on the currency outside banks, which usually differs from the final series. The preliminary series is often corrected several times as the CNB receives more accurate information about cash in banks' vaults and other items used when calculating the currency outside banks. Thus, the forecast is done using an inaccurate data series. For example, in July and August 2005, the average difference between preliminary and final values was 22.5 million HRK.

If we find some regularity in the difference between preliminary and final data, we would use it to improve the quality of preliminary values (adjust the series), which would, in turn, improve the quality of the forecast. Figure 6 presents the differences observed on Monday and Friday.

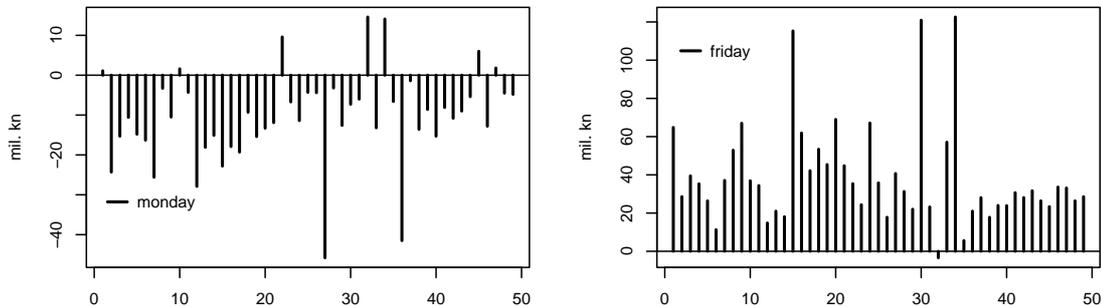


Figure 6: A difference between preliminary and final series for 2005 – Monday and Friday

From the Figure 6 it is obvious that the difference between final and preliminary values of the movement of currency is almost always negative on Monday and positive on Friday, which implies that the original data provided by banks constantly underestimate the final level on Monday and overestimate this value on Friday. These regularities were not observed on other weekdays (Table 3).

Commercial banks often correct the data about the amount of cash in their vaults they send to the CNB afterwards. Table 4 provides statistics of differences

and absolute differences of final series from the values received during the several subsequent days. The mark D1 refers to the difference between the final series and the originally received data, D2 to the difference between the final series and data received with a two day lag etc.

Table 3 Statistics of differences between preliminary and final values per day in 2005

2005	mon	tue	wen	thu	fri
<i>median</i>	-10,5	1,1	2	2,6	31,7
<i>mean</i>	-10,3	2,8	0,4	3,9	38,7
<i>std.dev.</i>	11,4	9,7	24	17,9	26,2

Table 4: Statistics of differences between preliminary and final values in 2005

	differences				Abs. differences			
	<i>mean</i>	<i>std.dev</i>	<i>min</i>	<i>max</i>	<i>mean</i>	<i>std.dev</i>	<i>min</i>	<i>max</i>
D1	7,1	25,2	-148,2	122,6	15,7	21,0	0,0	148,2
D2	3,4	21,2	-247,2	100,4	7,7	20,0	0,0	247,2
D3	0,8	19,3	-253,5	89,3	3,9	18,9	0,0	253,5
D4	0,3	18,4	-247,2	89,3	3,0	18,1	0,0	247,2
D5	1,1	8,7	-15,1	89,3	1,7	8,6	0,0	89,3
D6	1,2	8,6	-13,4	89,3	1,6	8,6	0,0	89,3
D7	1,2	8,5	-7,9	89,3	1,4	8,4	0,0	89,3
D8	1,0	8,5	-13,4	89,3	1,4	8,5	0,0	89,3
D9	1,1	8,5	-13,4	88,9	1,4	8,5	0,0	88,9
D10	1,0	8,4	-13,4	88,9	1,3	8,3	0,0	88,9

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