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Incorporating Uncertainties into Economic Forecasts: an Application to Forecasting Economic Activity in Croatia

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Incorporating Uncertainties into Economic Forecasts: an Application to Forecasting Economic Activity in Croatia*

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Abstract

In this paper we present a framework for incorporating uncertainties into economic forecasts which includes the calculation, presentation and evaluation of density forecasts. Using the VECM proposed by Rukelj (2010) as the benchmark model, this framework is applied to forecasting economic activity in Croatia. Future and parameter uncertainties using stochastic simulations are considered in density forecasting with parametric and non-parametric approach in generating random errors. The resulting density forecasts are presented using fan charts, and evaluated by the Kolmogorov-Smirnov test of probability integral transforms of density forecasts. The main findings are: parametric and non-parametric approach yield similar results, incorporating parameter uncertainty results in a much wider probability bands of the forecasts and evaluation of density forecasts indicate better performance when only future, without parameter, uncertainties are considered.

Keywords: Economic forecasting; density forecasting; fanchart; stochastic simulations.

JEL Classification: C41; C5.

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1 Introduction

Economic forecasting has been one of the most recurrent topics for policy makers. The failures and inaccuracies in forecasting witnessed across time has led to a long debate on predictive power and forecasts evaluation and the use of econometric tools when analyzing macroeconomic behavior and its projections has been revisited many times. As noticed by Garrat *et al.* (2003), in general macroeconomic forecasts are presented as point forecasts, and if considering uncertainty, the later is characterized by confidence intervals. This critic however dates long ago. Juster (1966) for example notices that a probability variable into consumer's surveys could predict more accurately purchase rates if compared to the predictors risen from a point projection of buying intentions. Although quite specific, the problem of accuracy in predicting the demand for durable goods by using such surveys is addressed in a way that concludes fitting in a more general issue in statistics: whether to incorporate probabilities as part of the forecasts or remaining with point predictions. In fact, Zarnowitz and Lambros (1982) compare consensus (point prediction) and uncertainty (diffuseness of the corresponding probability distributions), aiming to answer whether the dispersion of the point forecasts reflected lack of confidence of the correspondent predictor. As recently pointed by Engelberg *et al.* (2009), the incorporation of probabilities in forecasting can improve the interpretation of point predictions.

If literature has reached a consensus, that is uncertainties have to be incorporated into the forecasting framework. However this is not an easy task, as depending on the source of uncertainty (about the future, the parameters of the considered model, or the model itself) the forecaster has to evaluate different ways of taking each type of uncertainty into account. Probability forecasts using macroeconomic models for the US proposed by Fair (1980) accounted for uncertainty about the error terms, exogenous variable forecasts, the parameters and the possible misspecification of the model. It also considers the fact that the variances of forecast errors are not constant across time. Doan *et al.* (1984) chose the Bayesian approach in estimating a unrestricted and time-varying vector autoregressive processes for its forecasting exercise, showing that it improves the out-of-sample forecasts relative to univariate equations (by basically reporting a weighted likelihood by priors, unlike the unweighted likelihood coming from other alternatives), and explaining how the model could be used to make conditional projections and to analyze policy alternatives. More generally, Clements and Hendry (1995) examine frameworks for economic forecasting. Basically, the authors noticed that in the forecasting context, the methods for forecasting models and procedures might not be the only source of failures, but also the states of nature related to the properties of the variables to be forecast. As they argued that the assumptions of constant, time-invariant and stationary data generating processes, which were also thought as coincident with a unique model of the economy, were a wrong way or representing the world.

Practically, it has been observed that as many countries started their policies of inflation targets, the incorporation of the uncertainty in economic variable predictions served to show that there is uncertainty about shocks to affect the economy and about the nature of the transmission mechanism; it also helped to communicate with as minimum ambiguity as possible the views of the economic policy authority; and to have a better understanding of the sources of uncertainty (Blix and Sellin, 1998).

Methods to incorporate uncertainty are not unique. As shown by Britton *et al.* (1998), which proposed model is the one followed by the Bank of England for inflation and output growth forecasts, the early ranges of uncertainty in projections based on historical forecast errors were not as satisfactory as it was expected to be because by construction did not allow for asymmetries, thus not considering alternative scenarios and not enabling to conclude about the risk views. By being centered, the forecast would be representing upper and lower boundaries, rather than probabilities. Given this drawback, the Bank's forecasting method started considering probability distributions by assuming it had a known functional form and evaluating a limiting number of alternative assumptions, and this is what nowadays it is published by the monetary authority in the UK. Another example of a practical use of density forecasting is the Sveriges Riskbank's forecasting method, which is based on Blix and Sellin (1998), whose emphasize the important implications of slightly changing the model proposed by Britton *et al.* (1998). The change consists on determining the balance of risks by the subjective assessments of the macroeconomic variables of which uncertainty is considered.

In this paper, we consider the approach proposed by Garrat *et al.* (2003) regarding uncertainty forecasting, to be applied in forecasting economic activity in Croatia. The incorporation of uncertainty about the unobserved future shocks on forecasts and robustness of forecasts to the choice of the parameters is done for a given model, which is in our case the proposed by Rukelj (2010). That since the choice and estimation of an econometric model should accounts for the macroeconomic features of a given economy, so as it allows to base the forecasts upon it. In this sense, we believe a model such as in Rukelj (2010) balances economic theory and consistency with data, and so it is viable in forecasting.

The reminder of this paper is as follows: in the next section, we present the methodology. Incorporation of uncertainty in both future, and future and parameters are explained, and parametric and nonparametric approaches for stochastic simulation of errors are presented together with risk asymmetries. We present the results of density forecasts calculation and its performance evaluation in section 3, concluding in section 4.

2 Methodology

In this section we summarize the steps we followed to forecasting economic variables from a underlying general model that describes the main macroeconomic features of a given economy, taking uncertainty into consideration. The incorporation of uncertainty is carried out with two approaches: uncertainty about the future, and about the future and the parameters.

When uncertainty is about the future, the generation of probability forecasts is done in the absence of parameter uncertainty recursively by stochastic simulations. Accounting for the shocks in the forecasts is the critical point in density forecasting where a choice of shocks' distribution might be required according to the different types of uncertainty taken into account. Parametric and nonparametric methods are used in generation of shocks and the probability bands are constructed from the obtained set of simulated values. Shocks by parametric method from skewed distribution to account for asymmetries are also considered.

Uncertainty about the parameters also makes use of both parametric and nonparametric methods for the generation of the shocks. To generate density forecasts comprising parameter and future uncertainty, first the set of new in sample values of the variables is calculated by stochastic simulations using the original benchmark model and generated set of shocks; then, the model is reestimated for each replication of in sample generated series; finally, future uncertainty is taken into account for each reestimated model. Robustness checks were made by considering different number of replications in constructing probability bands.

Once calculations for shocks and probability bands are made, we evaluate the goodness of fit using the Kolmogorov - Smirnov test. In what follows, we present in detail each of the aforementioned steps.

2.1 Under Future Uncertainty

Let us consider the data generating process to be VAR model as follows:

$$y_t = \sum_{i=1}^{p} \Pi_i y_{t-i} + \pi_0 + \pi_1 t + e_t \tag{1}$$

where y_t is a vector of endogenous variables, Π_i is a matrix of coefficients to be estimated and π_0 and π_1 are vectors of deterministic terms. The term e_t is assumed to be a serially uncorrelated independent and identically distributed vector of shocks with zero means and positive definite covariance matrix.

Being ϕ_i the set of coefficients to be estimated, let us denote its estimates by $\hat{\phi}_i$, for all $i = 1, ..., n$. When estimating (1) for a finite sample of size T and recovering these estimates, the point forecasts in the period h are given by:

$$\hat{y}_{T+h} = \sum_{i=1}^{p} \hat{\Pi}_i y_{T+h-i} + \hat{\pi}_0 + \hat{\pi}_1(T+h) \tag{2}$$

where $t = 1, 2, ..., T$.

The stochastic simulation of the values of the series of interest h periods ahead is carried out via:

$$\hat{y}^{(x)}_{T+h} = \sum_{i=1}^{p} \hat{\Pi}_i y^{(x)}_{T+h-i} + \hat{\pi}_0 + \hat{\pi}_1(T+h) + u^{(x)}_{T+h} \tag{3}$$

where x refers to x^{th} replication of simulation algorithm. The term $u^{(x)}_{T+h}$ can be drawn using parametric or nonparametric approaches (discussed bellow).

Next, the probability bands are defined by threshold values such that:

- b^h is a vector of $\hat{y}^{(x)}_{T+h}$ values where $b(1)^h < b(2)^h < \ldots < b(r)^h$ where r is the number of replications;
- $\theta(b^h, p)$ is a function which gives the highest (θ^{μ}) and the lowest (θ^l) value of $\hat{y}^{(x)}_{T+h}$ for which it is estimated that there is a $p\%$ of probability for elements of $\hat{y}^{(x)}_{T+h}$ to be at period $T+h$, centered over the value $b(\frac{r}{2})^h$;
- $\theta^u(b^h, p) = b^h(i)$ where $i = \frac{r}{2}(1+p)$, and $\theta^l(b^h, p) = b^h(j)$ where $j = \frac{r}{2}(1-p)$, for $0 < p < 1$.

2.2 Under Future and Parameter Uncertainty

In this case, we use a bootstrap procedure to simulate s in sample values of y_t such that:

$$\hat{y}^{(s)}_t = \sum_{i=1}^{p} \hat{\Pi}_i y^{(s)}_{t-i} + \hat{\pi}_0 + \hat{\pi}_1 t + u^{(s)}_t \tag{4}$$

where $t = p+1, p+2, ..., T$.

It follows estimating the VAR model s times to obtain $\hat{\Pi}^s$, $\hat{\pi}_0^{(s)}$, $\hat{\pi}_1^{(s)}$ and $\hat{\Sigma}^{(s)}$, where $\hat{\Sigma}^{(s)}$ is the residuals' covariance matrix of the s^{th} estimated model. For each of these estimated models, r replications of the forecasts for the period $T+h$ are calculated as:

$$\hat{y}_{T+h}^{(x,s)} = \sum_{i=1}^{p} \hat{\Pi}_i^s y_{T+h-i}^{(x,s)} + \hat{\pi}_0^s + \hat{\pi}_1^s(T+h) + u_{T+h}^{(x,s)} \tag{5}$$

Similar to the previous case, the probability bands are defined by threshold values such that:

- b^h is a vector of $\hat{y}_{T+h}^{(x,s)}$ values where $b(1)^h < b(2)^h < \ldots < b(rs)^h$;
- $\theta(b^h, p)$ is a function which gives the highest (θ^{μ}) and the lowest (θ^l) value of $\hat{y}_{T+h}^{(x,s)}$ for which it is estimated that there is a $p\%$ of probability for elements of $\hat{y}_{T+h}^{(x,s)}$ to be at period $T+h$, centered over the value $b(\frac{rs}{2})^h$;
- $\theta^u(b^h, p) = b^h(i)$ where $i = \frac{rs}{2}(1+p)$, and $\theta^l(b^h, p) = b^h(j)$ where $j = \frac{rs}{2}(1-p)$, for $0 < p < 1$.

2.3 Simulated Errors

In the parametric approach, the errors are drawn from an assumed probability distribution function, which is in our case a multivariate normal distribution with mean zero and covariance matrix $\hat{\Sigma}$. The simulated errors are obtained as:

$$u_{t+h}^{(r,s)} = \hat{P}^{(s)} v_{t+h}^{(r,s)} \tag{6}$$

where $\hat{P}^{(s)}$ is the Cholesky decomposition of covariance matrix $\hat{\Sigma}$, such that $\hat{\Sigma}^{(s)} = \hat{P}^{(s)}\hat{P}^{(s)\prime}$ and $v_{t+h}^{(r,s)} \sim IIN(0, I)$.

On the other hand, the non-parametric procedure consists of generating the simulated errors as rs random draws with replacements from the in sample residuals $u_t^{(s)}$.

2.4 Unbalanced Risks

If unbalanced risks of the shocks are in the focus of analysis, asymmetries in forecasting should be considered. To account for these asymmetries, we take shocks obtained with the parametric method from a skewed distribution, such that the residuals are now defined as:

$$u_{t+h}^{(r,s)} = \hat{P}^{(s)} v_{t+h}^{(r,s)\prime} \tag{7}$$

where $\hat{P}^{(s)}$ is the Cholesky decomposition of covariance matrix $\hat{\Sigma}$, such that $\hat{\Sigma}^{(s)} = \hat{P}^{(s)}\hat{P}^{(s)\prime}$ and $v_{t+h}^{(r,s)\prime}$ is generated by two-piece standard normal distribution as follows[1]:

[1]See Tay and Wallis (2000) for more details.

$$f(v_{t+h}^{(r,s)\prime}, \mu, \sigma_1, \sigma_2) = \begin{cases} C \exp\left\{-\frac{1}{2\sigma_1^2}(x-\mu)^2\right\}, \text{for } x \leq \mu \\ C \exp\left\{-\frac{1}{2\sigma_2^2}(x-\mu)^2\right\}, \text{for } x > \mu \end{cases} \tag{8}$$

where $C = k(\sigma_1 + \sigma_2)^{-1}$ and $k = \sqrt{\frac{2}{\pi}}$. Skewness is described by $\gamma = \bar{\mu} - \mu = k(\sigma_2 - \sigma_1)$, where $\bar{\mu}$ is the mean and μ is the mode of distribution. The resulting distribution is unknown.

2.5 Forecasts evaluation

The forecasts evaluation are carried out with a Kolmogorov - Smirnov test of Probability Integral Transform (PIT) of outturns. This is a fully non-parametric test for comparing two (or more) distributions, hence it is robust as it does not rely on the location of the mean like the t-test, furthermore it does not depend on the underlying cumulative distribution function being tested. Unlike chi-square goodness-of-fit test, the K-S test does not depend is an adequate sample size for the approximations to be valid, it is an exact test[2]. In our case, after computing a sequence of one step ahead probability forecasts (with and without allowing for parameter uncertainty) for the considered simple events set out above over the the analized in sample period, the associated PIT for the benchmark model are found. Namely, density forecasts provide the complete probability distribution of possible variable outcomes in the forecasting horizon. Therefore, each outcome can be associated with the least or equal probability implied by density forecasts that will be actually observed by PIT's definition. The PIT associates the outturn in the observed period with the probability implied by the density forecasts that this outturn will be equal or less than actually observed. Good density forecasts would result in a uniformly distributed PIT of outturns. The hypothesis that these PIT are random draws from a Uniform distribution $U(0,1)$ is then tested by calculating the Kolmogorov-Smirnov statistic, such that large values are indicative of significant departures of the sample cumulative density function from the hypothesized uniform distribution.

3 Results

The benchmark model to be considered is a slightly modified version of the VECM proposed by Rukelj (2010)[3], as follows:

[2] Some important limitations of this test are: (1) it only applies to continuous distributions; (2) it tends to be more sensitive near the center of the distribution than at the tails; and (3) the distribution must be fully specified, i.e., if location, scale, and shape parameters are estimated from the data, the critical region of the K-S test is no longer valid. It typically must be determined by simulation. Due to limitations 2 and 3 above, many analysts prefer to use the Anderson-Darling goodness-of-fit test. However, the Anderson-Darling test is only available for a few specific distributions.

[3] As mentioned in the previous section, we chose the model proposed by Rukelj (2010) as it accurately accounts for Croatian's macroeconomic features, and it performes well when forecasting.

$$\Delta x_t = \alpha(\beta' x_{t-1} + \hat{b}_1(t-1) + \hat{c}) + \hat{\Gamma}_1 \Delta x_{t-1} + \ldots + \hat{\Gamma}_6 \Delta x_{t-6} + u_t \tag{9}$$

where x_t is vector of endogenous variables (y, g and m), $\hat{b_1}$ is a vector of estimated deterministic trend coefficients and $\hat{c}$ is a vector of constants. Rewritting (9) in a VAR form:

$$x_t = \sum_{i=1}^{7} \hat{\Pi}_i y_{t-i} + \hat{\pi}_0 + \hat{\pi}_1 t + e_t \tag{10}$$

where $\hat{\Pi}_i = I_3 - \alpha\beta' + \hat{\Gamma}_1$, $\hat{\Pi}_i = \hat{\Gamma}_i - \hat{\Gamma}_{i-1}$, $i = 2, .., 6$ and $\hat{\Pi}_7 = -\hat{\Gamma}_6$.

Up to 1000 forecasts under future uncertainty are recovered as in (3), both for parametric and nonparametric approaches of generating simulated errors, whereas under future and parameter uncertainty the 1000 samples are extracted with the following procedure: (i) generate initial values for the first 7 lags of the series of interest; (ii) calculate forecasts ahead by the estimated parameters of the initial model, as well as applying a shock to each observation in each period through parametric or non parametric method; (iii) as a result, there will be 1000 samples with three variable for which 1000 models are estimated; (iv) based on this 1000 models, forecasts are made like under only future uncertainty. For the determination of the probability bands, we sort the different forecasts by deciles, i.e., the 10% probability that the forecasts will fall within the 0th and the 100th observations, the 20% probability of falling within the 0th and the 200th observations, and so on. Naturally, the closer the forecast is to the 500th observation, the narrower the bands are.

We apply the previous described methodology to forecasting economic activity in Croatia on monthly basis from 1997:01 to 2008:12 available from public sources expressed in real terms using 1997 as a base year and seasonal adjusted[4]. The results are shown in fugures 1 to 5 in the appendix[5]. The charts show the considered possible outcomes. As described in the previous section, the forecasted variable would be within the central band with 10% of probability, and will get to the wider bands (lighter regions) with additional 10% chances each time (being the wider bands representative of more uncertain situations). From figures 1 to 3, the density forecasts suggest the economic activity index would have increased the first trimester of 2009, regardless the method for errors simulation. A considered skewed distribution such that $\sigma_1 = 1$ and $\sigma_2 = 2$ would increase the chances of observing a higher index onwards.

[4]The economic activity index is constructed as a proxy of national output as it is described in Rukelj (2010) since GDP data is available only on a quarterly basis.

[5]Vertical axis: logarithm of Croatian economic activity index, actual and forecasted. Number of replications: 1000.

However, the actual levels of the economic activity index during 2009 showed an important difference with what the model results are if only taking into consideration future uncertainty. That is due to a decrease in 8% of the economic activity itself, which reached in mid 2009 levels bellow the observed in the beginning of 2006. Although the forecasts are not accurate in this period, it is worth to mention that Croatia experienced its historically greatest economic shock in 2009. In the context of this paper, such shock would have seen as if outturn was pushed to the last decile of the density forecast. When adding parameter uncertainty, it is possible to check that increases and decreases of the index would be similarly probable even when considering unbalanced risks (see figures 4 and 5).

Next, we examine the out of sample PIT outturns relative to fan chart probability distributions by forecasting horizon from 2005:1 to 2008:12. For this, we estimated 36 models taking a initial sample from 1997:01 to 2005:01, and increasing it in one month at each time, until completing the forecasting horizon. Then, density forecasts for the each model in the following 12 month period were produced and the PIT of outturns with respect to these density forecasts were calculated. Charts 6 to 10 summarize the results: the black dots represent the PIT of the outturns in each period of the forecasting horizon and their positions on faded regions indicate in which percentile of the fan chart each outturn fell in at each forecast horizon. Thus, satisfactory results would be seen as if the outturns were disperse among probability bands. As it is shown, this is true when considering only future uncertainty (see figures 6 to 8), whereas incorporation of parameter uncertainty make all outturns concentrate around the median, indicating a poor performance of this second model.

Finally we check whether the performance of the different considered models was satisfactory. The next table summarizes the results of the K-S test of goodness of fit. It indicates that there is no evidence at 5% significance level to reject the null hypothesis that outturns were drawn from the uniform distribution in most of the periods when only future uncertainty is considered. Regardless of loss function, we could say that the correct density is weakly superior to all forecasts, which suggests that we evaluate forecasts by assessing whether the forecast densities are correct.

Kolmogorov (D)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
					Future uncertainty, parametric							
Value (D)	0.14	0.20	0.24	0.18	0.30	0.26	0.21	0.24	0.22	0.22	0.21	0.19
Adjusted value	0.83	1.25	1.47	1.13	1.83	1.60	1.30	1.49	1.34	1.38	1.26	1.17
Probability	0.49	0.09	0.03	0.16	0.00	0.01	0.07	0.02	0.06	0.04	0.08	0.13
					Future uncertainty, non - parametric							
Value (D)	0.17	0.22	0.26	0.22	0.29	0.28	0.24	0.28	0.25	0.24	0.22	0.20
Adjusted value	1.06	1.33	1.62	1.37	1.79	1.69	1.49	1.73	1.52	1.45	1.37	1.24
Probability	0.21	0.06	0.01	0.05	0.00	0.01	0.02	0.00	0.02	0.03	0.05	0.09
					Future uncertainty, skewed (s1=1, s2=2)							
Value (D)	0.20	0.17	0.19	0.17	0.23	0.29	0.26	0.30	0.30	0.27	0.29	0.27
Adjusted value	1.22	1.06	1.16	1.04	1.39	1.79	1.60	1.86	1.86	1.67	1.79	1.66
Probability	0.10	0.21	0.14	0.23	0.04	0.00	0.01	0.00	0.00	0.00	0.00	0.01
					Future and parameter uncertainty, parametric							
Value (D)	0.41	0.42	0.41	0.43	0.40	0.42	0.39	0.38	0.38	0.37	0.38	0.37
Adjusted value	2.53	2.61	2.52	2.65	2.45	2.59	2.42	2.35	2.33	2.30	2.35	2.28
Probability	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
					Future and parameter uncertainty, non - parametric							
Value (D)	0.35	0.38	0.38	0.35	0.39	0.38	0.36	0.39	0.33	0.35	0.35	0.32
Adjusted value	2.18	2.35	2.35	2.15	2.42	2.33	2.22	2.39	2.01	2.15	2.15	1.96
Probability	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

4 Conclusions

Although the economic theory has rapidly developed sophisticated and reasonably well behave forecasting models, risks and uncertainty are recurrently present. Thus the need of taking these factors into account when forecasting is what motivated economists to incorporate probabilities to represent this degree of ignorance regarding future events that might affect the economy.

In this paper, we presented a framework to incorporate uncertainties into the forecasts of a economic activity index for Croatia. Using the VECM proposed by Rukelj (2010) as the benchmark model, we allowed the proposed framework to consider uncertainty in both future, and future and parameter. Furthermore, we provided a presentation of density forecasts using stochastic simulations of the random errors with parametric and non-parametric approaches, and the evaluation of density forecasts, the latter using Kolmogorov-Smirnov test of probability integral transforms. We found that forecasts of the economic activity index for Croatia are similar when incorporating future uncertainty, regardless the approach used for errors simulation. However, incorporating parameter uncertainty not only returns much wider probability bands, but also performs poorly regardless the approach used for errors simulation.

We believe our main contributions are: firstly, that we present density forecasts of economic activity for Croatia for the first time; secondly, we follow the most recent developments regarding this part of the literature and in addition introduce skewness in future distributions in order to represent potential risks in Croatian economy. The application of the provided forecasting approach could certainly contribute to transparency regarding the views of monetary and fiscal authorities. All the aforementioned - we expect - will initiate a important debate among economists and contribute to a important jump in developing and improving this forecasting approach for this economy. Future research we propose should incorporate model uncertainty and additional goodness of fit tests.

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A Fan Charts and PIT outturns

Figure 1: Fan Chart: Future Uncertainty, Parametric Simulated Errors, Symmetric Risks

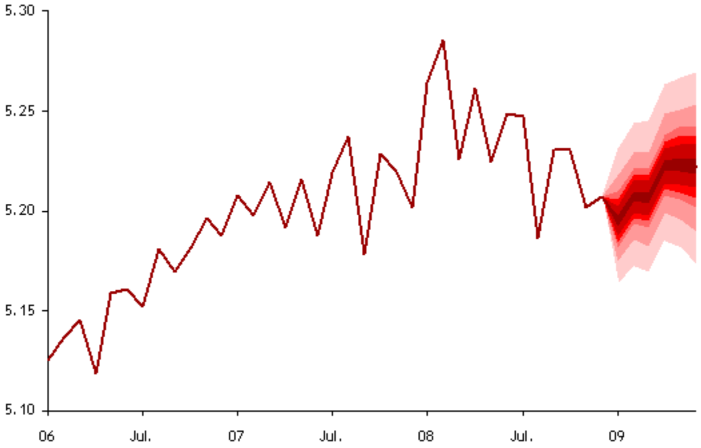

Figure 2: Fan Chart: Future Uncertainty, Non-Parametric Simulated Errors, Symmetric Risks

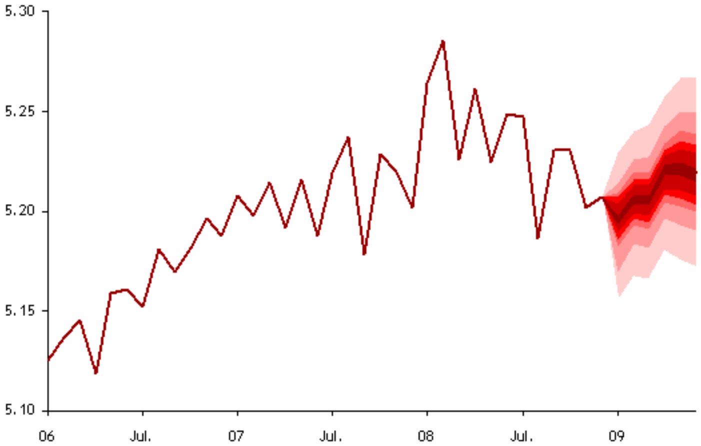

Figure 3: Fan Chart: Future Uncertainty, Parametric Simulated Errors, Asymmetric Risks

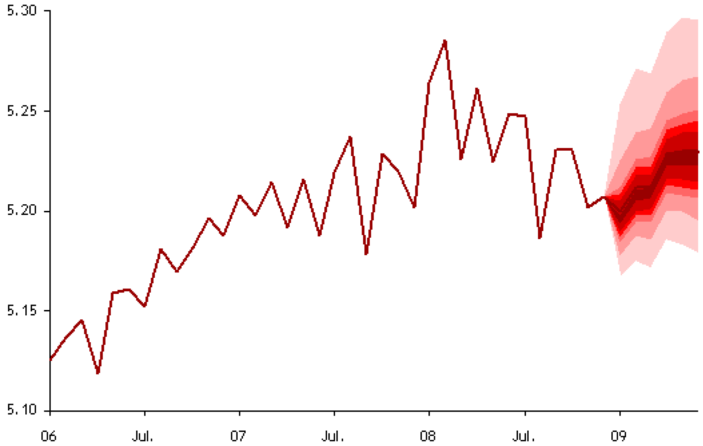

Figure 4: Fan Chart: Future and Parameter Uncertainty, Parametric Simulated Errors, Symmetric Risks

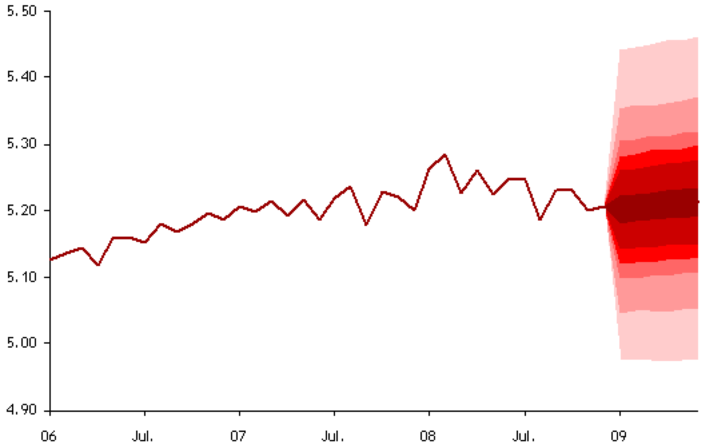

Figure 5: Fan Chart: Future and Parameter Uncertainty, Non-Parametric Simulated Errors, Symmetric Risks

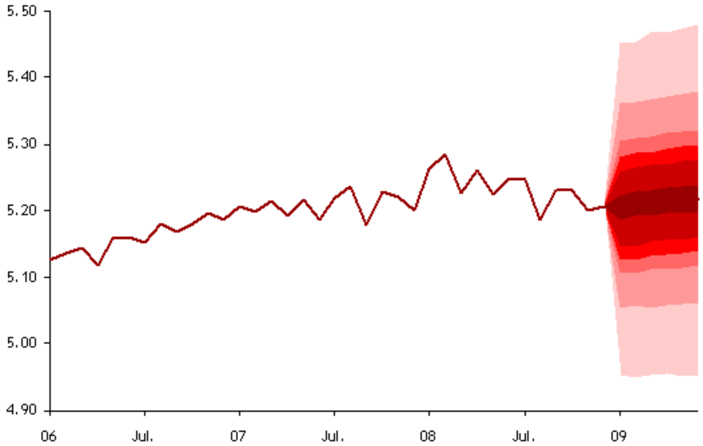

Figure 6: PIT outturns: Future Uncertainty, Parametric Simulated Errors, Symmetric Risks

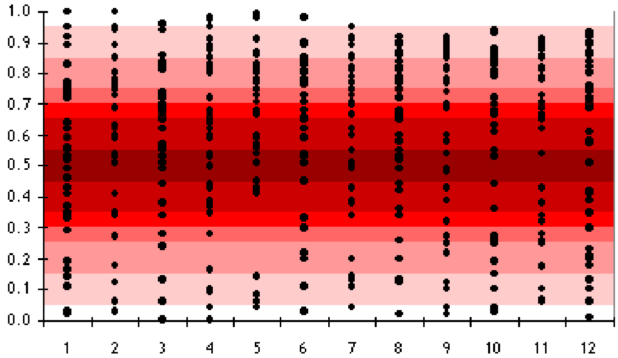

Figure 7: PIT outturns: Future Uncertainty, Non-Parametric Simulated Errors, Symmetric Risks

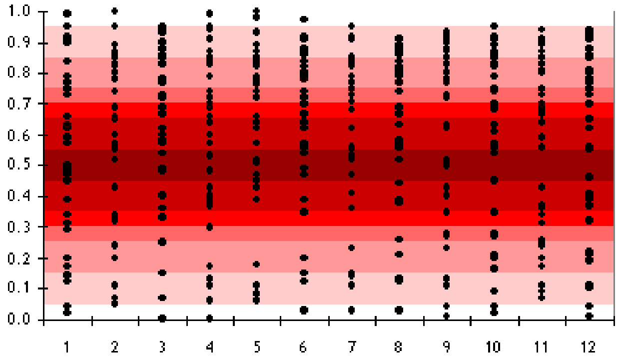

Figure 8: PIT outturns: Future Uncertainty, Parametric Simulated Errors, Asymmetric Risks

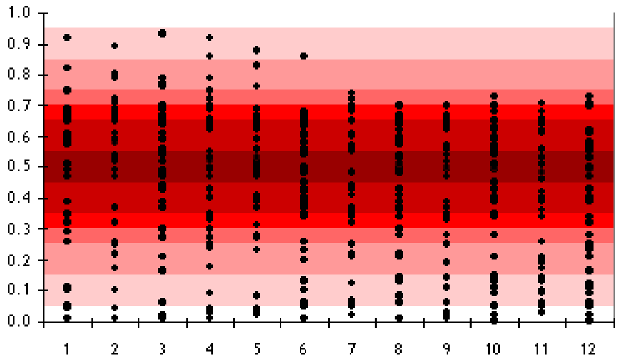

Figure 9: PIT outturns: Future and Parameter Uncertainty, Parametric Simulated Errors, Symmetric Risks

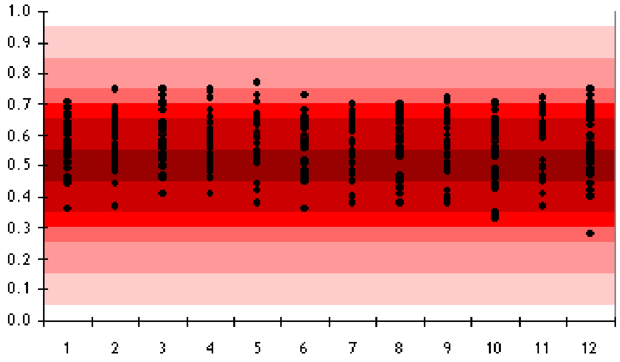

Figure 10: PIT outturns: Future and Parameter Uncertainty, Non-Parametric Simulated Errors, Symmetric Risks