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A Mathematical Model and Decision Support System for Determination of the Values of the Marginal Reserve Requirement as Instrument of Monetary Policy

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Abstract

This paper is a short version of my master theses, which I was defending in July 2007 on the Faculty of Economics, University of Zagreb.
It studies the real problem of interdependence between central bank and commercial banks goals in Croatia. The basic central bank task is to achieve and to maintain price stability. Croatian external debt has been increasing for years and so the activities of the Croatian National Bank are designed to correct this situation. In order to stop the further increase of the external debt, the Croatian National Bank uses several monetary policy instruments, among which are the marginal reserve requirement and the subscription of compulsory Croatian National Bank (CNB) bills. On the other hand, the goal of commercial banks is to maximise profits. Banks take loans from abroad at a lower interest rate and invest this money in Croatia at a higher interest rate, thus fulfilling their goal, but contributing to the external debt increase. In order to obtain the desired effects of the marginal reserve requirement, its optimal percentage value should be determined. This problem is modelled as a bilevel mixed 0-1 programming problem. The objective of the leader (Croatian National Bank) is to minimize the increase in household loans by setting different percentages of the reserve requirements for loans extended to households and for those granted to enterprises. The objective of the followers (banks) is to maximize profits. I consider two models of the problem. The first one is a model that covers the marginal reserve requirement and the special reserve requirement. The second model is the first one extended to cover the subscription of compulsory CNB bills. In order to solve this NP-hard problem a heuristic is proposed. In order to verify the model, the paper ends with simulations and the presentation of computational results.

Key words: monetary policy instruments, commercial banks credit activity, marginal reserve requirement, Croatian National Bank bills, new measure, bilevel mixed 0-1 programming problem, NP-hard problem, heuristic

1 INTRODUCTION

The Croatian external debt has been increasing for years. A large proportion of this foreign indebtedness of the Republic of Croatia is the foreign debt of the Croatian commercial banks. The owners of Croatian commercial banks are mostly located in EU. Since the interest rates on loans made at the European market are considerably lower than in Croatia, the EU banks have a great interest in buying Croatian banks which then take loans from abroad at a lower

interest rate and invest this money in Croatia at a higher interest rate. In this way the EU banks make a profit in a very easy way. Presently, the domestic demand for loans extended in Croatia is very high and EU banks are doing a good deal.

On the other side, the Croatian National Bank (CNB), within the framework of its monetary policy, carries out various measures to maintain price stability that is, to maintain the maximum amount of economic growth concurrently with a low rate of inflation, and thus fulfils its statutorily defined role. History tells us that periods of high inflation on the whole lead to economic collapse, since the attention of enterprises shifts from the production of goods to the preservation of the assets that they own and/or manage.

In an economy like Croatia, which makes more than half of its GDP in trade with the rest of the world, there is an additional danger from the import of inflation, and price stability also means stability of the exchange rate with the currency to which most credit operations are indexed, as well as the price of fixed assets (in the case of Croatia this is the euro). Similarly, because of the exceptional degree of openness of the market, there is a large chance that inflation can be transferred from abroad (for example, a large rise in the price of fuels on the world market will have a direct impact on the rise in domestic prices). For these reasons the total foreign debt of Croatia is important, since if an influx of money occur it would have a powerful impact on the stability of domestic prices, producing a major deterioration in the balance of trade of the country, which could lead to long-term problems in the development of the economy.

As it was mentioned before, since the foreign debt of the commercial banks constitutes a large portion of the total foreign debt of the Republic of Croatia, the CNB makes use of measures to affect the behaviour of the banks. One of them, which is particularly considered in this paper, is the marginal reserves requirement (MRR).[1] This is the obligatory foreign currency deposit placed for an indefinite period of time in a foreign currency account at the central bank. The amount of the deposit is defined in terms of a percentage of a base that is made up of the cumulative rise of foreign liabilities of the bank as against the base period. It is a considerable problem to determine the value of this percentage in order to achieve the desired effects on the banking system.

[1] In 2004 the application of MRR started, at a rate of 24%; in February 2005 this was increased to 30% and calculated according to the same base period. In May that year the rate was increased once again and came to 40% with respect to the initial calculation period. In November 2005 the decision was made in such a way that non-working days were included in the calculation, and the day of the calculation was shifted to the second Wednesday in the month. In December 2005 with a new change in the decision, a second calculation period was introduced, and for a growth in a debt with respect to this new calculation period an additional 15% of the growth in foreign borrowing had to be set aside. In the last change in the decision, June 2006, the base for MRR was extended to guarantees and other investments that are kept off-balance sheet, and the MRR rate for such loans is 55%.

Apart from the marginal reserve requirement, two other measures are presently used by the CNB, and these are the special reserve requirement (SRR)[2] and compulsory central bank bills (CCBB). The special reserve requirement is similar to the marginal reserve requirement, but differs from it in the commercial bank debt sources that it covers. It is calculated on bonds issued, irrespective of whether the bonds have been issued on the domestic or foreign market. Compulsory CNB bills are securities to which commercial banks must subscribe at the central bank when they exceed the allowed growth of loans made. The subscription is regulated by special central bank regulations[3], and the maturity period is 360 days. A particular feature of these securities is that they cannot be traded on the secondary market.

In the operations of the Croatian banks, on the liability side, a major part is constituted by short-term deposits, while another important source of funds consists of mid-term and long-term loans taken out on the world financial market. On the asset side, on the other hand, mid-term and long-term loans made to households and enterprises prevail. There is an increasingly important share of short-term loans to the household sector via overdraft facilities and credit cards. Since the banks have been lending to the household sector to a larger extent in recent years, with households taking on debt to a much greater extent for final consumption, mainly of imported goods, the CNB measures are endeavouring to persuade the banks to reduce loans made to the household sector for the sake of final consumption. In this manner the CNB endeavours to affect the whole of the economy, since the behaviour of banks and households, had there been no central bank reaction, would certainly have led to an economic breakdown.

For the purpose of determining the percentage of MRR, this paper formulates a mathematical model of the interdependence of central bank and commercial banks activity. The model belongs to the class of bilevel mixed 0-1 programming models. At the upper level there is central bank decision making, and according to the framework of this paper, its objective is to minimise the growth in loans extended to households by the commercial banks. On the lower or following level is the decision making of commercial banks, the goal of which is to maximise profit. The profit of commercial banks is defined as the difference in the interest that they charge on loans extended in the domestic market and the interest they have to pay for foreign borrowings. Other revenues of commercial banks are in this paper ignored and considered only as a mean to cover operational costs. The model has been developed with given premises that simplify the real situation, but on the assumption of professional implementation it could without any essential inroads into the basic idea be developed with other elements of the relation between the central bank and the commercial banks. Two models are observed. The first and basic model covers the MRR and the special reserve requirement (SRR). The second model is an extension of the first model for the subscription

[2] Special reserve requirement started in the first quarter of 2006, after the last change of the rate of the marginal reserve requirement. From the beginning it was started at a rate of 55%.

[3] Compulsory CNB bills were brought into force the first time in 2003, and then the allowed growth of loans was 16% per annum, or 4% per quarter, and their subscription took place at the same tempo. The measure was in force again from January 2007, the initial allowed growth of loans being 1% per month. But during the year this measure has been changed several times. Unlike 2003, when the measure lasted only a year, it is expected that this measure will now last some considerable time.

of compulsory central bank bills (CCBB). The SRR is included in the first model only at the level of definition, while in the case of numerical simulations the issue of bonds by the commercial banks is ignored. This assumption does not distort the applicability of the model since on the one hand it does not affect the numerical results and on the other hand immediately after its introduction the commercial banks halted their borrowings via the issue of bonds.

The paper consists of five chapters. In the second chapter the mathematical formulations of two mentioned models are given and explained. The problems are modelled as bilevel mixed 0-1 programming models. Since the problems of bilevel programming belong to the class of NP-hard problems, heuristics have been developed for solving them. The heuristics developed for the purposes of this paper, and the basic methods used for the making of these heuristics and the relaxations, are described in the third chapter.

In the fourth chapter the results of numerical simulations obtained by solving with the help of the heuristics described in the preceding chapter are given. Analyses are made with the use of so called effective interest rates. These interest rates are calculated according to the CNB regulation and take into account all the loan's costs. These rates are the real loan's prices. At the end, in the fifth chapter, conclusions are given and suggestions are made for further research for the purpose of developing the model.

2 MATHEMATICAL MODELS

The essential characteristic of all bilevel programming problems is that the decision maker at the upper level (the leader) should affect the lower level decision makers (the follower), the follower's possible choice and the benefit that derives from this choice.

Here we are looking at a large hierarchical system and the process of interaction that goes on in time, where the leader is the central bank and the follower is the set of all the commercial banks. Each commercial bank acts independently from the other commercial banks, in line with its own business strategy.

From a mathematical point of view, the problem of bilevel programming is an NP-hard problem. Hence in practice heuristics are created, which is the method used in this paper.

This two models have been defined that describe the contemporary situation in the working of the central bank vis-à-vis the commercial banks. The first model covers MRR and SRR. The second model, which describes the current state of affairs, is an extension of the first model by the subscription of compulsory CNB bills.

The reason for division of the contemporary situation into the two models lies in the fact that the subscription of compulsory CNB bills is a measure that has been in force since January 1, 2007. An additional reason for the split into two models is the fact that this is a measure that is considered a short-lasting instrument, which will be in force for only a limited period of

time.[4] Resources that are bound by investment in compulsory CNB bills are repaid into the system when the bill matures.[5] On the other hand, in the reserve requirement large resources with no maturity date are accumulated, which cannot at any given time be put back into the system without consequences being incurred.

In the models it is assumed that the central bank objective is the reduction in the growth of commercial bank loans to the household sector. The decision the central bank has to make is what the percentage of the MRR and SRR are. The maximisation of profit is assumed to be the objective of the commercial banks.

2.1 A mathematical model that covers the marginal reserve requirement and the special reserve requirement

For the sake of formulating the said mathematical model, I make the following definitions:
Indices

- i - type of borrowing (i=1 loans taken from abroad, i=2 bonds)
- j - commercial bank, $j = 1. ..., J$
- l - type of loans of borrowed resources (l=1 housing loans, l=2 other loans to households, l=3 loans to enterprises)
- p - prescribed percentage of marginal / special reserve requirement
- t - time for which debt is taken on (macro period), $t = 1. ..., T$
- τ - time for which loans are made (micro period) $\tau \in S_t$,, $t = 1. ..., T$

Parameters

- op[6] - percentage of reserve requirement
- kb - commercial bank credit multiplier
- x_{jil0} - stock of debt of bank j from source of resources i, that it has loaned in loans l at the beginning of the period under observation
- W_{jl0} - stock of loans l of bank j at the beginning of the period under observation
- o_l - number of instalments for repayment of the loan l
- b_i - number of payments for the return of the borrowed sum i
- k_{it} - interest rate at which money is borrowed from source i in period t
- m_{jlt} - interest rate at which bank j loans resources l in period t
- d_{lt} - minimum demand that has to be met for loans l in period t
- g_{lt} - maximum supply of loans l in period t

[4] When this measure was previously in force it lasted for a year only, that is, during 2003, the calculation and setting aside started at the end of the first quarter of 2003, and ended at the end of the first quarter in 2004.

[5] The current decision prescribes a maturity period of 360 days.

[6] $op = 17\%$.

Notations

y_{jilpt} - amount of resources that bank j has repaid via annuity payments for sources of resources i, at interest rate k_{it} with depositing marginal reserve requirement at the rate of p in period t, which it has loaned in loan l

W_{jlt} - total amount of resources that bank j has loaned in loan l in the macro period t at the interest rate of m_{jlt}

U_{jlt} - total amount of resources that bank j has received in annuities from loan l at an interest rate of m_{jlt} in the macro period t

Q_{jipt} - stock of debt of bank j from source i borrowed at an interest rate of k_{it}, along with depositing marginal reserve requirement at a rate of p at the end of the period t

R_{jlt} - stock of loan of bank j in loan l loaned at an interest rate of m_{jlt} at the end of period t

Decision variables

x_{jilpt} - amount of resources that bank j borrowed in period t from source i, at interest rate k_{it} in conjunction with depositing for marginal (special) reserve requirement at a rate of p, in order to lend them in loan l

$w_{jl\tau}$ - amount of resources that bank j lent in loan l in micro period τ at an interest rate of m_{jlt}

$$z_{ilpt} = \begin{cases} 1, & \text{if the percentage of prescribed marginal (i = 1)/ special (i = 2) reserve} \\ & \text{requirement on resources borrowed for loan l in period t is equal to p} \\ 0, & \text{otherwise} \end{cases}$$

l = 1, 2 (loans to households), l = 3 (loans to enterprises)

$$v_{jilpt} = \begin{cases} 1, \text{if } \sum_{i,p,t} (x_{jilpt} - y_{jilpt}) > 0 \\ 0, \text{otherwise} \end{cases}$$

The following expressions hold for the notations

$$y_{jilpt} = \frac{1}{b_i}(x_{jil0} + \sum_{\alpha=\max(t-b_i,1)}^{t-1} z_{ilp\alpha} x_{jilp\alpha}) \ , \quad \forall\, j,i,l,p,t \qquad \text{(a)}$$

$$W_{jlt} = \sum_{\tau \in S_t} w_{jl\tau} \qquad \forall j,l,t \qquad \text{(b)}$$

$$U_{jlt} = \frac{1}{o_l}(W_{jl0} + \sum_{\alpha=\max(t-o_l,1)}^{t-1} W_{jl\alpha}) \ , \ \forall\, j,l,t \qquad \text{(c)}$$

$$Q_{jipt} = \sum_{l=1}^{3}(x_{jil0} + \sum_{\alpha=1}^{t}(x_{jilp\alpha} - y_{jilp\alpha}))\ , \qquad \forall\, j,i,p,t \qquad \text{(d)}$$

$$R_{jlt} = W_{jl0} + \sum_{\alpha=1}^{t}(W_{jl\alpha} - U_{jl\alpha})\ , \qquad \forall\, j,l,t \qquad \text{(e)}$$

Expression (a) shows how much resources bank *j* has to return in each period on the basis of the loans it has taken out abroad or on the basis of bonds it has issued.[7] Expression (b) shows the total loans of bank *j* in macro period *t*, [8] and expression (c) shows the total receipts of the same bank on the basis of repayment of the part of the loans made in the previous periods.[9] Expression (d) shows the total stock of indebtedness of bank *j* at the end of period *t*,[10] while expression (e) tells of the stock of the loans of bank *j* and the end of period *t*.[11]

The model:

$$\min_{z} \sum_{\substack{l=1,2 \\ j,t}} (W_{jlt} - U_{jlt}) \qquad \text{(I)}$$

Subject to the constraints:

$$\sum_{p} z_{ilpt} = 1, \qquad \forall i,l,t \qquad (1)$$

$$\max \sum_{t} \left(\sum_{l} m_{jlt} R_{jlt} - \sum_{i,p} k_{it} Q_{jipt} \right) \qquad \forall j \qquad \text{(II)}$$

[7] It is assumed that resources are repaid in equal monthly instalments. In reality this is not the case, but as a rule the banks repay their borrowings in quarterly, semi-annual or annual instalments, while they usually pay out bonds on maturity, with the interest (the coupon) being paid semi-annual or annual, while there are also variants, such as the whole amount (principal and interest) being paid out only at maturity.

[8] The amount of intercalary interest that the bank collects for the payment of loans before the date of maturity is negligible compared with all the other revenues of the commercial banks.

[9] Today banks grant loans with flexible maturity dates (they can be any day in the month). Earlier the maturity dates were fixed for all clients and as a rule the maturity of a loan would be the first or last day in the month, irrespective of the day the loan was granted.

[10] Bank liabilities.

[11] Bank assets.

Subject to the constraints:

$$W_{jlt} \le \left(\left(kb\left(\frac{100-op}{100} \right) \right) \left(\sum_i x_{jil0} + \sum_{i,p,\alpha=1}^{t} v_{jilpt} \frac{100-p}{100} z_{ilp\alpha} (x_{jilp\alpha} - y_{jilp\alpha}) \right. \right.$$
$$\left. \left. + \sum_{i,p,\alpha=1}^{t} (1 - v_{jilpt})(x_{jilp\alpha} - y_{jilp\alpha}) \right) \right) + \sum_{\alpha=1}^{t} U_{jl\alpha} - \sum_{\alpha=1}^{t-1} W_{jl\alpha} - W_{jl0} \qquad (2)$$
$$\forall j,l,t$$

$$x_{jilpt} \le M z_{ilpt} \ , \qquad \forall j,i,l,p,t \qquad (3)$$

$$\sum_{\alpha=1}^{t} (x_{jilp\alpha} - y_{jilp\alpha}) \le M v_{jilpt} \ , \qquad \forall j,i,l,p,t \qquad (4)$$

$$\sum_{\alpha=1}^{t} (y_{jilp\alpha} - x_{jilp\alpha}) \le M(1 - v_{jilpt}) , \qquad \forall j,i,l,p,t \qquad (5)$$

$$W_{jlt} \ge d_{jlt} \ , \qquad \forall j,t,l = 1,2 \qquad (6)$$

$$W_{jlt} \le g_{jlt} \ , \qquad \forall j,l,t \qquad (7)$$

$$x_{jiplt},\ w_{jl\tau} \ge 0, \quad z_{jlpt},\ v_{jilpt} \in \{0,\ 1\}, \quad \forall\ j,i,l,p,t \qquad (8)$$

In expressions (a) and (c) the amounts are equal to the repayment quotas (instalments), while revenue made from interest collected is used to cover interest expenses and for operational profit made.

For the credit multiplier of the commercial bank (*kb*) the expression $kb = \frac{1}{1 - sz \times (1 - op)}$ holds, where *op* is the prescribed percentage of the reserve requirement, and *sz* is the percentage of the borrowing retained in the bank in the form of the loan it makes. With an assumed rate of retention *sz* = 60%, then:

$$kb = \frac{100}{100 - 0.6 \times (100 - op)} = \frac{100}{40 + 0.6op}, \qquad (9)$$

By introducing expression (9) into (2) I obtain the expression:

$$W_{jlt} \le \left(\left(\frac{100 - op}{40 + 0.6op} \right) \left(\sum_i x_{jil0} + \sum_{i,p,\alpha=1}^{t} v_{jilpt} \frac{100 - p}{100} z_{ilp\alpha} (x_{jilp\alpha} - y_{jilp\alpha}) + \sum_{i,p,\alpha=1}^{t} (1 - v_{jilpt})(x_{jilp\alpha} - y_{jilp\alpha}) \right) \right) + \sum_{\alpha=1}^{t} U_{jl\alpha} - \sum_{\alpha=1}^{t-1} W_{jl\alpha} - W_{jl0} \quad (10)$$

$\forall j, l, t$

In model (I)-(10) the central bank minimises the growth of loans to households, while the commercial banks, within the framework of the constraints that are forced on them by the upper level (the leader), maximise their profits. The micro and macro periods are included into the model because the central bank (leader) makes its decision about changes in the rates of the marginal or special reserve requirements once a month, while the followers (the commercial banks) can make their decisions daily.

Constraints (1) say that the percentages of marginal and special reserve requirement can take exactly one value for every kind of source of resources and for each kind of loan by the banks. They are the same for all the banks and are independent of the interest rates at which the bank obtains the resources. Apart from that, these percentages are always integer values.

Constraints (10) show that the resources that bank *j* can extend in instrument *l* cannot be greater than the resources available. The available resources of bank *j* consist of the initial stock of the borrowed resources to which the remaining resources borrowed from sources covered by the marginal or special reserve requirement are added, minus the resources repaid for previous borrowings, minus the percentage of the reserve if the bank's borrowing rises, or only diminished by this amount if the borrowing of the bank from the sources of resources covered falls. These resources are multiplied by the credit multiplier in the period under observation. Collected repayments of loans made earlier diminished by resources extended in previous periods also form part of the available resources.

Constraints (3) describe the fact that banks can borrow abroad only at one rate of the prescribed marginal (i=1) / special (i=2) reserve requirement on resources borrowed for loan *l* in period *t* where *M* is a large real number.

Constraints (4) and (5) are essential for the determination of the status of the binary variable *v* that says whether in some period there are overall more borrowed or repaid resources,[12] which is essential for the sake of setting aside the marginal or special reserve requirements.

Constraints (6) are the lower bound for every type of loan of every bank in every period. The lower bound is the level of planned market share or the threshold of profitability.

Constraints (7) show that the maximum demand for resources is known[13] in the case of every bank in every period that can be met, that is, the upper bound to the level of loans made.

And at the end, constraints (8) are the constraints of non-negativity and binarity of the variables.

[12] Depending on whether the total foreign borrowing is rising or falling.

[13] In reality this means that the bank has set a maximum amount of loans in each period with which it will satisfy the shown demand for loan resources.

2.2 The mathematical model extended to cover the subscription of compulsory CNB bills

This model differs from the preceding model only in the constraints (10), which have been replaced by the following constraints:

$$W_{jlt} \le \left(\left(\frac{100-op}{40+0{,}6op} \right) \left(\sum_i x_{jil0} + \sum_{i,p,\alpha=1}^{t} v_{jilpt} \frac{100-p}{100} z_{ilp\alpha} (x_{jilp\alpha} - y_{jilp\alpha}) \right. \right.$$
$$\left. \left. + \sum_{i,p,\alpha=1}^{t} (1 - v_{jilpt})(x_{jilp\alpha} - y_{jilp\alpha}) \right) \right) + \sum_{\alpha=1}^{t} U_{jl\alpha} - \sum_{\alpha=1}^{t-1} W_{jl\alpha} - W_{jl0} \qquad \forall j,l,t \qquad (11)$$
$$- 0.5 \sum_{\alpha=2}^{t} \left(\max \left\{ \left(\frac{\sum_{\beta=1}^{\alpha-1} W_{jl\beta}}{W_{jl0}} - \frac{\alpha-1}{100} \right), 0 \right\} \cdot W_{jl0} \right)$$

These constraints show the maximum available resources that bank *j* can invest in instrument *l*. The available resources of bank *j* consist of the initial stock of the borrowed funds to which are added remaining resources borrowed from sources covered by the marginal or special reserve requirements diminished by repaid resources from earlier borrowing, and diminished by the percentage of this reserve if bank borrowing should increase, or diminished only by this amount if the borrowing of the bank in terms of sources of resources covered should fall. These resources are multiplied by the credit multiplier in the observed period. Also counted in available resources are collected repayments of loans made earlier diminished by loans made in the previous periods. Should a commercial bank exceed the allowed growth of loans, the available resources will be diminished by the amount that the bank has to use to subscribe compulsory CNB bills.

3. RELAXATION OF THE MODEL AND HEURISTICS FOR PROBLEM SOLVING

The problem described, and all bilevel programming problems belong to the class of NP-hard problems. For the solution of these problems, heuristics have been developed based on search methods. The heuristics are conducted according to this procedure:

1. For the given bank, the rate of the marginal/special reserve requirement is changed, and it is checked whether making loans is profitable at this level, according to existing interest rates, for each kind of loan.
2. The lowest and highest rates of reserve at which operations case to be valuable for each kind of loan separately are memorized.
3. If the solution for each kind of loan is found, then the search procedure is stopped.
4. A check is made to see if all the banks have been considered. If they have not, the next bank is chosen, and step 1 is repeated; if they have, then we pass to step 5.
5. A calculation is made of the average marginal/special reserve requirement for the whole system in terms of each kind of loan separately.
6. Stop of the calculation process.

The flow diagram for the constructed heuristics is shown in Fig. 1. In all models constructed within the context of this paper, the same heuristic is used, but with a differently relaxed model.

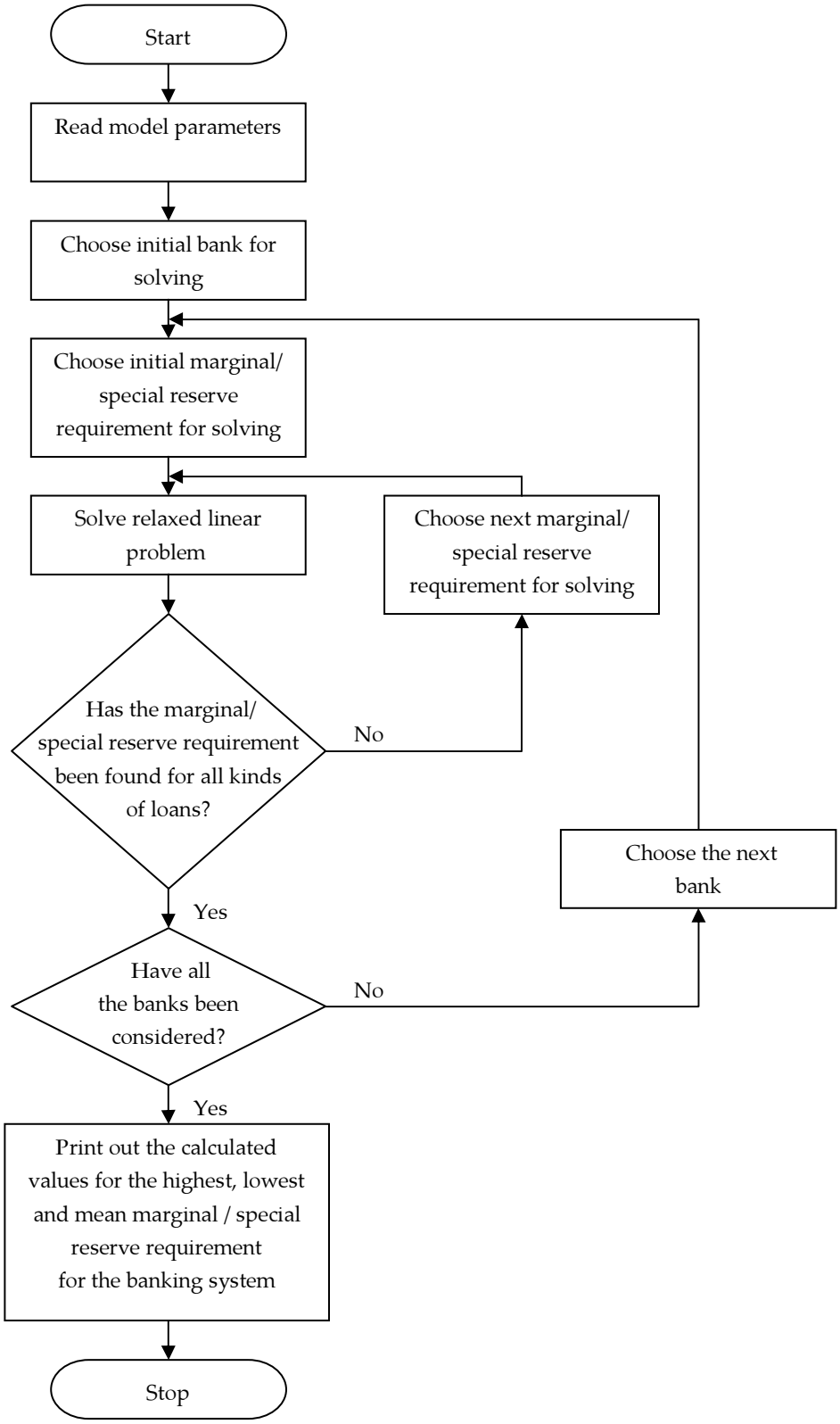

Figure 1. Heuristics flow chart.

In the numerical part, certain relaxations have been carried out to simplify the model, or for the sake of confidentiality, so as not to let the data of the bank used be identified as coming from that bank.

The first assumption is that the interest rate on foreign borrowings is the same for all banks and comes to EURIBOR + 1.5 percentage points for borrowing and EURIBOR + 2 percentage points for bonds. The average value of EURIBOR for borrowings for one year used in the numerical calculation is 4.06%.

The interest rate at which the banks extend their funds ranges from 4.12% to 9.68% for housing loans. In other loans to the household sector the interest rates range from 7.76% to 17.6%. Interest rates for loans to the enterprises range from 5.84% to 15.33%.[14] As I stated before, these interest rates are not the nominal rates, but the rates calculated according to the CNB regulation taking into account all the loan's costs.

As stated in the description of the model, the rate of marginal and special reserve requirements is always an integer value. The analysis was carried out in a range from 10% to 80%, for in this case, together with the reserve requirement (which according to the official decision on setting aside the reserve requirement comes to 17%), it means that the maximum to be set aside for reserves is 97% of the amount of the increase of borrowed sources of funds, which are the subject of the research. A further assumption is that the rates of the marginal and special reserve requirements are equal ($z_{1lpt}= z_{2lpt}$) and that the percentage of marginal/special reserve requirement on all kinds of loans made is equal ($z_{i1pt}= z_{i2pt}= z_{i3pt}$).
Only macro periods are observed in the model, since the sum of all events in the micro periods within a single macro period can be seen as a single event during this macro period.
Since the lower (market share of the bank) and the upper (maximum demand) bounds are defined in the model for every kind of loan, I observed the following circumstances from the numerical calculations. For each bank and each kind of loan, in the case of low rates of marginal reserve requirement, the funds lent out are on the upper bound. As the rates of marginal reserve requirement gradually increase, certain loans fall from the upper bound to the lower bound (that is, from the amount sufficient to satisfy the maximum demand they decline to an amount sufficient to achieve the planned market share). For each bank and each kind of loan there is one rate of marginal reserve requirement at which the funds extended drop from the upper to the lower bound. Such rates are then calculated for each bank and each kind of loan separately. Then, the average value of MRR as the arithmetical mean of all MRRs is calculated, for each kind of loan made, and not just the minimum and maximum. The average value is calculated as the arithmetical mean because I assumed that market shares are identical. It should be said that this relaxation was done so that the banks in the analysis should not be identifiable from the market share and interest rates data. The decision support system created allows for various market shares without any problems or efforts. In addition,

[14] Data obtained from CNB and used in this paper. Certain categories constitute averages of several different types of loans (kuna loans, foreign currency loans, etc.)

if it is taken into account that most of the interest rates are the same for a given kind of loan and a given bank, market share does not actually play a great part during the calculation of the average value of MRR.

In the framework of this heuristic the value of the binary variable v_{ijlpt} was also set at 1[15] in all points of observation. In other words, it is assumed that there is a constant rise in bank borrowing, since this corresponds to the real life situation in the banking system. It is possible to replace borrowing from loans via bonds and vice versa. And that is why in the numerical calculations, without loosing generality, I calculated only MRR.

The amount of the marginal/special reserve requirements is observed, at which amount of a given kind of loan moves from the upper to the lower bound of lending. The leader (the central bank) prescribes the rate of marginal/special reserve requirements (p), while the commercial banks then decide about the amount they will borrow under the conditions set (they cannot affect the upper level or the world capital market).

At the beginning an equal market share is assumed for all banks, and in the calculations of the average values of the rates of marginal/special reserve requirements it is enough to use the ordinary arithmetical mean. The initial level of loans from which the calculation starts for each individual bank is done in such a way that the calculation is carried out with a rate of marginal reserve requirement of 55%, and then the aggregate of all loans extended is distributed according to the proportions of given loans in total loans made by the banking sector. There are two types of commercial banks, distinguished by whether they do or do not give housing loans. The shares of the loans made depending on the type of bank are given in the table below.

Table 1. Shares of loans in the total loans of a commercial bank (in %)

Type of loan	Housing loans	Other loans to households	Loans to enterprises
Banks that do give housing loans	31.87	32.45	35.68
Banks that do not give housing loans	0,00	47.63	52.37

Source: CNB

In linear programmes that helped in the calculation the expressions for the notations were removed. The values that are given by the notations from the first model are calculated later in the computer programme. The only notations used are those for the returns of loans made earlier (y_{jilpt}).

[15] $v_{jilpt} = 1$, $\forall\, j,i,l,p,t$

4. NUMERICAL RESULTS AND ANALYSES

Banks in the Croatian banking system are observed. The basic difference among the different banks is the difference in the interest rate policy. The values of interest rates are obtained from the CNB. The average annual values of each individual bank are taken.

The amount of profit in conjunction with varied rates of marginal reserve requirement is observed, that is, the rate of marginal reserve requirement at which given loans fall from the upper to the lower bound.

Table 2. Shares of loans in the banking system (in %)

Type of loan	Housing loans	Other loans to households	Loans to enterprises
Proportions of loans	24.37	36.02	39.61

Source: CNB

With the help of data concerning the proportions of each kind of loans inside the banking system and the calculated average values for the marginal reserve requirement within each individual model, the rate of marginal reserve requirement that has a major effect on system behaviour was calculated.

I note that the level of loans made is limited, which means that there is an upper and a lower bound. For a small MRR, each bank, in each period, for each type of loan, makes its loans maximally, and if the MRR is large, then minimally. The threshold at which the change described in the behaviour of a commercial bank occurs varies according to the individual commercial bank.

Tables 3 – 5 give the minimum and maximum values of MRR at which in terms of each kind of loans and individual banks there is a change in behaviour, i.e., a fall from the upper to the lower bound of loans made (the threshold of profitability is reached or the planned market share for this kind of loan in this bank has been achieved). Then a calculation is made of the average value of the threshold of MRR as arithmetical mean of all MRR thresholds.

In this way in the case of the model that covers MRR and SRR, Table 3 is obtained:

Table 3. Results of the basic model of the current state of affairs (in %)

Type of loan	Minimum MRR	Maximum MRR	Average MRR
Housing loans	18	65	44.62
Other loans to households	57	80	67.71
Loans to enterprises	42	78	62.29

Source: authors' calculations

By calculating from data concerning the average rate of MRR impact, the rate of 59.94%[16] was obtained as the MRR that had an impact on the system.

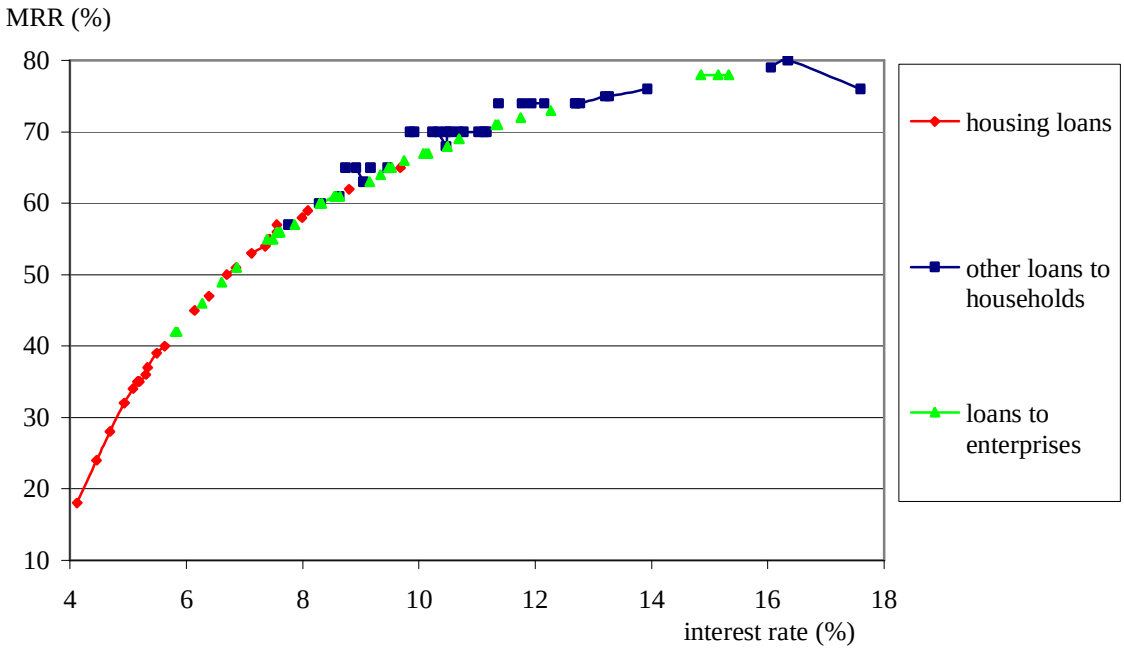

Figure 2. The impact of change in MRR on the profitability of loans depending on the interest rate – basic mathematical model

After that, an analysis was carried out of the model extended to the subscription of compulsory CNB bills and the following results were obtained for this:

Table 4. Results of the model extended to the subscription of compulsory CNB bills (in %)

Type of loan	Minimum MRR	Maximum MRR	Average MRR
Housing loans	18	65	44.23
Other loans to households	44	76	60,65
Loans to enterprises	42	78	61.97

Source: authors' calculations

For this model, the rate of 57.17% was calculated as the rate at which the MRR had an impact on the system. According to expectations, it was lower than the rate from the previous model.

[16] 0.2437*44.62%+0.3602*67.71%+0.3961*62.29%=59.94%

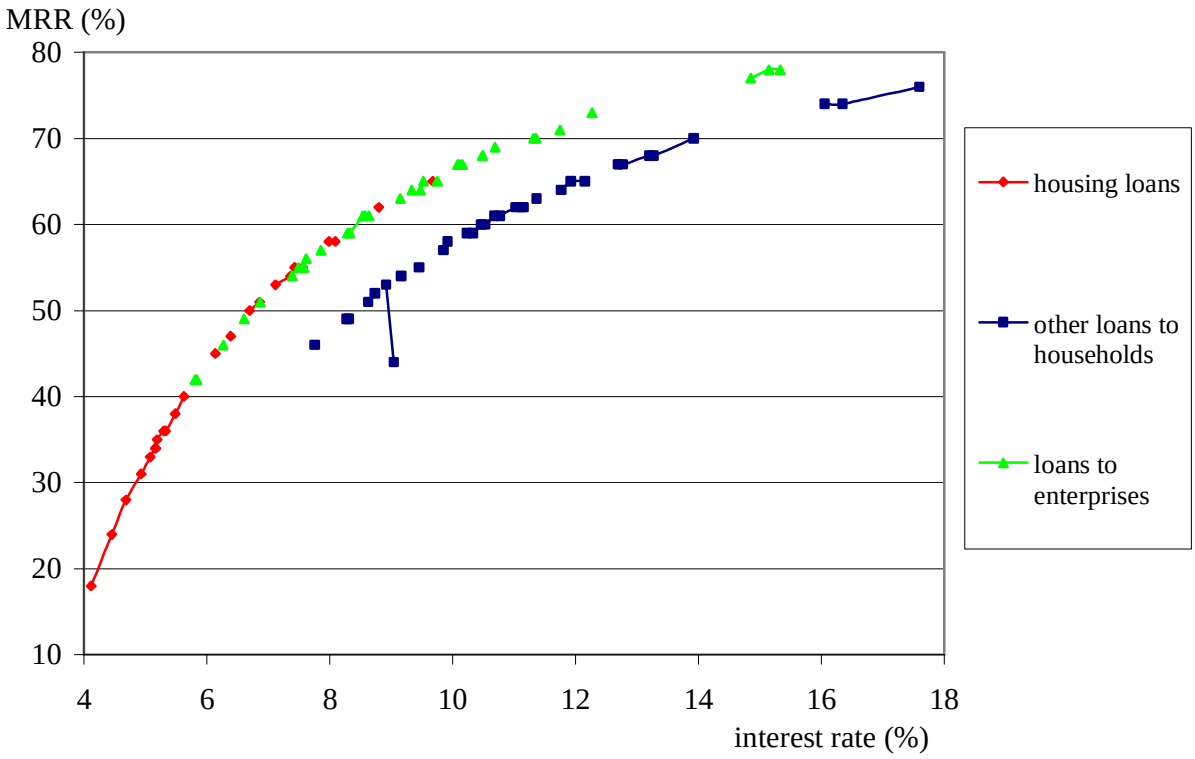

Figure 3. The impact of a change of MRR on the profitability of loans depending on the interest rate – the mathematical model extended to the subscription of compulsory CNB bills

Graphs about the impact of a change in MRR on operational profitability depending on the interest rate show an interesting feature, which is the divergence of the line of impact on other loans to households from the main line of MRR impact, in the case of the model extended to the subscription of compulsory central bank bills (Figure 2 and Figure 3).

An interesting fact that we can see from the table of the results (Table 4) is that in the model that covers the subscription of compulsory CNB bills, the average rate of the impact of MRR on other loans to households is lower than the average rate of impact on loans to enterprises. From the graph it can also be seen that the instrument of subscription of compulsory CNB bills has almost no effect on housing loans, or on loans to enterprises. One explanation of this might be that these loans of the banks are already at a sensibly lower level of profitability than the other loans to households, and thus the impact of the fall in profitability is sensibly lower. Apart from that, the models allowed a certain kind of loan to stay at the initial level, and this has an impact on the results of the analyses carried out with the use of the mathematical models created.

If the statistical data of the central bank covered loans of central banks that are hidden from statistical monitoring and made at high interest rates, it is highly probably that the rates of MRR impact calculated would be still higher.

5. CONCLUSION

The graphs show us that within the framework of all the models that even when the MRR rate is changed, the MRR has an equal effect on housing loans and loans to enterprises, a change in impact existing only in the case of other loans to households.

Commercial banks run the kind of business (interest rate) policy such that the rate of MRR prescribed by the central bank does not have negative effect on the level of loans, but only on their profitability. If, for example, housing loans are advanced at a low interest rate, this will be made up for by the high interest rate on loans to enterprises or other loans to households. Some other banks will advance all loans according to a unified interest rate. Because of market conditions, the commercial banks do not put up interest rates on current banking products very significantly, so as not to lose market share; rather, they endeavour to bring in new products that include various charges and fees, and in this way practically raise the interest rate.

The Republic of Croatia has a relatively high foreign debt, exceeding 80% of GDP. This is a level of debt that is considered a potential trigger of a debt crisis. The growth of the debt is considerably influenced by commercial banks, borrowing abroad, because of the insufficiency of domestic capital accumulation, in order to lend these funds via various kinds of loan to households and corporations.

The objective of the CNB is defined by law, and this is, primarily, price stability, in other words, the stability of the whole of the economic system of the Republic of Croatia. The central bank, through its measures, can affect only the banking system, and is endeavouring through these measures to hold back the growth in bank foreign debts.

The mathematical models of bilevel programming developed in this paper describe only the relations between central bank and commercial banks oriented to the foreign borrowings of the commercial banks. The main goal of the central bank is to reduce the growth of loans from banks to the household sector, since enterprise entities can, with relative ease, take on debt abroad directly, which the central bank can not affect. The results of the analysis of these models show that the use of the decision support system created, would be useful in an ex-ante analysis of the effects of monetary policy. Although the analysis uses assumptions that simplify the model, the results are still a good signpost of the way the central bank might proceed while making decisions about the level of the MRR. Every additional fact related to the Croatian monetary system, and real data, can very easily be included into the decision support system, which would thus have an important role in the regulation of monetary trends. In the forthcoming period it would be important to improve the central bank statistics monitoring the loans of commercial banks. Data obtained in this way could give a much better picture of the banking sector of the Republic of Croatia. With the help of data so processed, the central bank would be able to adopt more precisely targeted measures with respect to a certain kind of commercial bank behaviour.

5.1. GUIDELINES FOR FUTURE RESEARCH

In forthcoming research changes might be made in the existing model of calculating the rate of impact of the marginal/special reserve requirements by having this rate calculated for each bank separately according to the distribution of its loans. A rate calculated for each individual commercial bank, along with the rate of the impact of the marginal/special reserve requirements of the system, would be added according to the bank's market share. This would naturally affect the relaxation of the model in which I assumed that borrowing abroad increases all the time (i.e., that all the binary variables were fixed at $v_{jilpt} = 1$, and in further research it would be necessary to hypothesize the possible fall in borrowing).

It would be necessary to carry out an analysis of the sensitivity of the optimal solution to a change in parameters, i.e., for various commercial bank strategies expressed in the changing interest rate or in the accomplishment of the planned market share to observe the impact on the marginal reserves. Certain other functions of the objectives of the central bank and the commercial banks can also be observed. It is possible to observe certain sub-problems as being multicriterion, where the objectives would be profit and accomplishment of market share.

One direction for new research might be other measures that the central bank might prescribe which in the current conditions might halt further growth of the foreign debt of the banking sector of the Republic of Croatia.

REFERENCES

[1] Babic M. (1996): "Makroekonomija", MATE d.o.o., Zagreb

[2] Beenstock M., Azoulay E., Offenbacher A., Sulla O. (2001): "A macroeconomic model with oligopolistic banks: monetary control, inflation and growth in Israel", Economic Modeling 20 (2003) 455-486

[3] Candler W., Townsley R. (1982): "A linear two-level programming problem", Computers and Operations Research 9 (1982) 59-76

[4] Deng X. (1998): "Compexity issues in bilevel linear programming", In A. Migdalas, P.M. Pardalos and P. Värbrand, editors, Multilevel Optimization: Algorithms and Applications (1998) 149-164, Kluwer Academic Publishers, Dordrecht

[5] Ellison M., Valla N. (2000): "Learning, uncertainty and central bank activism in an economy with strategic interactions", Journal of Monetary Economics 48 (2001) 153-171

[6] Galac T., Dukic L. (2005): "Rezultati četvrtoga HNB-ova anketiranja banaka", Hrvatska narodna banka, Zagreb

[7] Garey M.R., Johnson D.S. (1979): "Computers and Intractability: A Guide to the Theory of NP-Completeness", W.H. Freeman and Co., San Francisco

[8] Kahn M., Kandel S., Sarig O. (2001): "Real and nominal effects of central bank monetary policy", Journal of Monetary Economics 49 (2002) 1493-1519

[9] Moore J.T., Bard J.F. (1990): "The Mixed Integer Linear Bilevel Programming Problem", Operations Research, Vol. 38 (1990), No. 5, 911-921

[10] www.hnb.hr

[11] Pongrac D. (2007): "Modeli međuovisnosti djelovanja centralne banke i poslovnih banaka i heuristike za njihovo rješavanje", magistarski rad, Sveučilište u Zagrebu, Ekonomski fakultet